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The debt to wealth ratio vs the debt to GDP ratio as an indicator of financial stability

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The debt to wealth ratio vs the debt to GDP ratio as an indicator of financial stability

Valeria De Bonis

Abstract. The application of the SGP fiscal criteria has evolved over time. This paper reviews the changing role of the debt benchmark in the light of Carlo Casarosa's analysis of the economic meaning of the public debt to GDP ratio.

Keywords: SGP fiscal criteria; debt to GDP ratio; wealth to GDP ratio; EU governance; Italy.
JEL Classification: H63, H77, F45.

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1 INTRODUCTION

The application of the European fiscal rules has been suspended after the outburst of the Covid pandemic and the most recent events related to the Ukrainian war. In the public debate, the restoration of the Stability and Growth Pact should be accompanied by a reform of the fiscal rules to adapt them to the profoundly altered economic environment and to overcome some of their drawbacks. The way that the EU will decide to take will be of particular importance for Italy that, in the ranking of countries with the highest debt/GDP ratio, is second only to Greece. Actually, in the public debate some agreement is emerging on making the debt to GDP ratio the anchor of the new framework. The appropriateness of this variable as an indicator of financial stability is, however, questionable. From a theoretical point of view, growth models point at wealth as the pivot variable; form a practical one, the debt to GP ratio alone is not a good predictor of sovereign crises. Casarosa (1986) had already criticised the rationale underlying the use of the debt to GDP ratio, based on the argument that the debt to (the demand for) wealth is the proper indicator of the financial situation of the public sector.

Against this background, the paper proceeds as follows.

Section 2 discusses the economic meaning of the debt to GDP ratio vis-à-vis the evolution of the wealth to GDP ratio along Casarosa's (1986) contribution. Section 3 looks at the evolution of the debt to GDP ratio in Italy in the last 15 years, confronting it with dynamics of the wealth to GDP and debt to wealth ratios and looks at an indicator of fiscal sustainability based on a normalisation of variables using private wealth instead of GDP. Section 4 concludes the

paper in the context of the reform proposals of the European fiscal rules.

2 THE 'LAW OF CHANGE' OF THE WEALTH TO GDP RATIO

The debt to GDP ratio is one of the most commonly referred to indicators of the sustainability of public finances. The recent literature – see, among others, Balassone *et al* (2007), Giammarioli *et al* (2007), Wyplosz (2011), Furman and Summers (2020), Blanchard et al (2021) - has, however, questioned its appropriateness. Among the critiques, there is the inconsistency of the comparison of a stock and a flow variable. Furman and Summers (2020) propose to substitute this indicator either with the ratio of debt to the present value of GDP (two stock variables) or with the ratio of interest payments (real interest expenditure, calculated as interest expenditure minus the product between the inflation rate and the debt stock) to GDP (two flow variables).

Casarosa (1986) had already critically analysed the economic rationale underlying the use of the debt to GDP ratio. Based on the argument that the debt to (the demand for) wealth is the proper indicator of the financial situation of the public sector, the paper raises two major points:

- the debt to GDP ratio is a good proxy of the debt to wealth ratio only if the debt to wealth ratio remains constant; otherwise, the debt to GDP ratio and its changes do not deliver clear implications as for the sustainability of public debt, since one can witness an increase of the debt to GDP ratio with a contemporaneous decrease of the debt to wealth ratio;
- the wealth to GDP ratio tends to an equilibrium level that is a function of the average growth rate; this implies that, if the latter decreases, the former increases and its actual level converges slowly to it; as a consequence, the debt to GDP ratio can rise without any negative implication on the financial situation of the public sector.

The use of the debt to GDP ratio is, thus, a shortcut, consisting in the application of the steady state results to the dynamic analysis.

In fact, if one assumes a constant propensity to save and a constant natural rate of growth of income, the steady state wealth to GDP ratio is constant; its value increases in the propensity to save and decreases in the income growth rate.

However, if the economy is off the balanced growth path, the wealth to GDP ratio can vary, increasing/decreasing continuously, even if the propensity to save and the natural growth rate of income remain constant.

To show this, Casarosa (1986) formulates the 'law of change' of the demand of assets w.r.t. GDP, stating that the wealth to GDP ratio increases/remains constant/decreases if the growth rate of the demand for assets is higher/equal/lower than the income growth rate. The income growth rate is given by the sum of the population growth rate, n, and the technical progress growth rate, m, exogenously given.

If the propensity to save, s, and the income growth rate, $(n+m)$, are constant, the equilibrium value of the wealth to GDP ratio, β, is:

$$\beta = \frac{s}{(n+m)} \qquad (1)$$

The steady state is stable: for instance, starting from the equilibrium position, an increase in s implies an increase in β, which converges to the new (higher) equilibrium level (and vice versa for a fall in s). The same applies to a permanent fall/increase in $(n+m)$.

As Casarosa (1986) shows, the speed of convergence is low; thus, the wealth to GDP ratio might well increase/decrease for a long time. Analogous results apply if the propensity to save is an increasing function of β.

The steady state equation (1) is an accounting identity. Though Casarosa's (1986) analysis is derived in a closed economy setting, it also applies to an open economy one. The fundamental accounting identities, however, must take into account the presence of exports and imports of goods and services, and of capital with the associated interest payments. It is therefore necessary to distinguish between production, that is, GDP, and income, that is, gross national product, GNP. Moreover, net foreign assets (f) become a component of wealth.

In the case of a small open economy with perfect capital mobility, one can show (see Appendix 1) that the ratio between wealth and income (GNP), γ, is given by the usual formula

$$\gamma = \frac{s}{(n+m)}, \qquad (2)$$

while the ratio between wealth and domestic production (GDP), β', is given by

$$\beta' = \frac{s\left(1-r\frac{k^*}{g(k^*)}\right)}{(n+m-sr)}, \qquad (3)$$

where $r\frac{k^*}{g(k^*)}$ is the domestic capital share and *r* is the world interest rate.

Dynamic optimising models offer another perspective showing the weakness of the debt to GDP ratio as an indicator of financial stability: their transversality conditions concern stock variables, among which wealth. Bruce and Turnovsky (1999) consequently normalise the outstanding stock of government debt and the present value of the government primary deficit in the expression for the government intertemporal budget constraint.

They also construct an indicator of fiscal sustainability expressed as the present value of fiscal policy adjustments, relative to the (initial) capital stock, necessary to ensure sustainability, that is, equality between the present value of the government primary deficit and the opposite of the value of the initial outstanding public debt stock.

Blanchard et al (1990) had previously proposed an analogous index, expressed as a flow, that is, relative to GNP: they consider the constant tax rate that would satisfy the equality between the present value of the government primary deficit and the opposite of the value of the initial outstanding public debt stock. The indicator of sutainability is then given by the difference in actual and future tax rates (Bruce and Turnowsky, 1999, consider, instead, lump sum taxes). The indicator can be associated to different time horizons to indicate the tax rate that would leave the debt to GNP ratio unchanged. For instance, the medium term gap, corresponding to a five-year time horizon, can be approximated by the difference between the average over the next five years of primary expenditure plus the initial debt to GNP ratio multiplied by the difference between the real interest rate and the growth rate. It should be noted that the above discussion about sustainability hinges on the assumption that the real interest rate exceeds the growth rate, otherwise there would be no need to run budget surpluses.

3 THE REAL CONTEXT

As pointed out by Casarosa (1986), in the real economy one can witness changes in (n+m) and s that make the equilibrium analysis meaningless. Nevertheless, the law of change is always valid, and it is therefore possible to determine the actual dynamics of the wealth to GDP ratio and therefore to contrast the changes in the debt to GDP and debt to wealth ratios.

In what follows, we look at Italian data for the period 2005-2021. Data for the debt to GDP ratio are taken from Eurostat (Government consolidated gross debt in percentage of GDP); as for wealth, the model of the previous section delivers an evolution of wealth stemming from saving flows; the actual value of wealth is, instead, determined also by the change of asset prices (capital gains). The *actual wealth* value (family and nonprofit sector) is taken from Banca d'Italia-Istat (2022) and is given by the difference between assets and liabilities values. Wealth values predicted on the basis of saving flows only are calculated using Eurostat data (gross household disposable income; gross and net household saving rates for *predicted wealth* and *net* - of capital depreciation - *predicted wealth*, respectively). As for the law of change, we look at the difference between the rate of change of wealth (in the three different specifications) and rate of change of GDP.

Figure 1 shows the evolution in the actual wealth to GDP ratio for Italy in the 2005-2020 period, while Figure 2 and Figure 3 show the evolution of predicted and predicted net of capital depreciation wealth.

Positive (negative) differentials correspond to increases (decreases) in the wealth to GDP ratio.

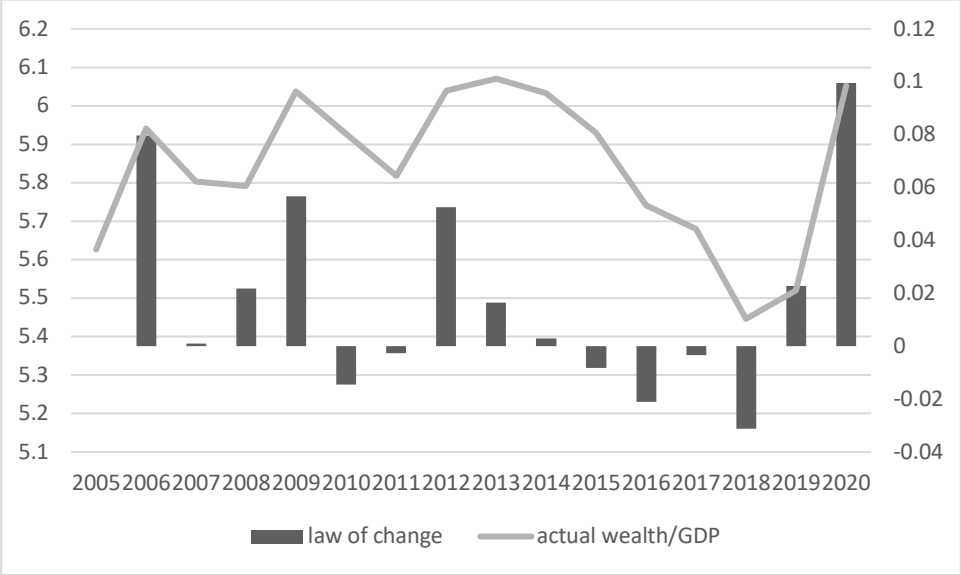

Figure 1 Actual wealth to GDP ratio: Italy 206-2020

As mentioned above, the value of the actual wealth variable is affected by changes in asset prices, particularly the decrease of housing prices beginning in 2013. However, also the specifications based on saving flows only show a constant decrease in the wealth to GDP ratio in the period 2013-2018. In Appendix 2 we add mandatory social security contributions, that constitute mandatory savings adding to social security wealth, to saving flows, obtaining a similar trend.

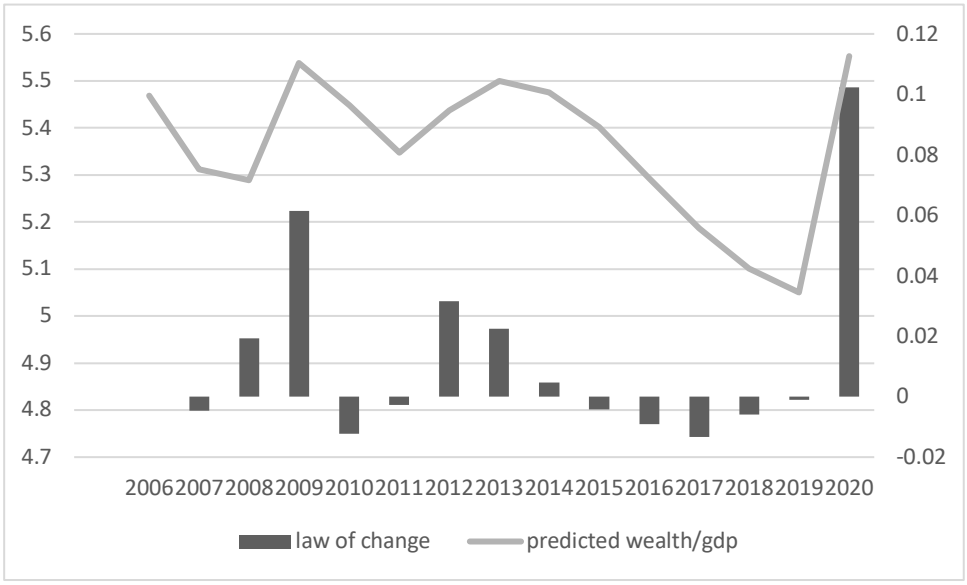

Figure 2 Predicted wealth to GDP ratio: Italy 2006-2020

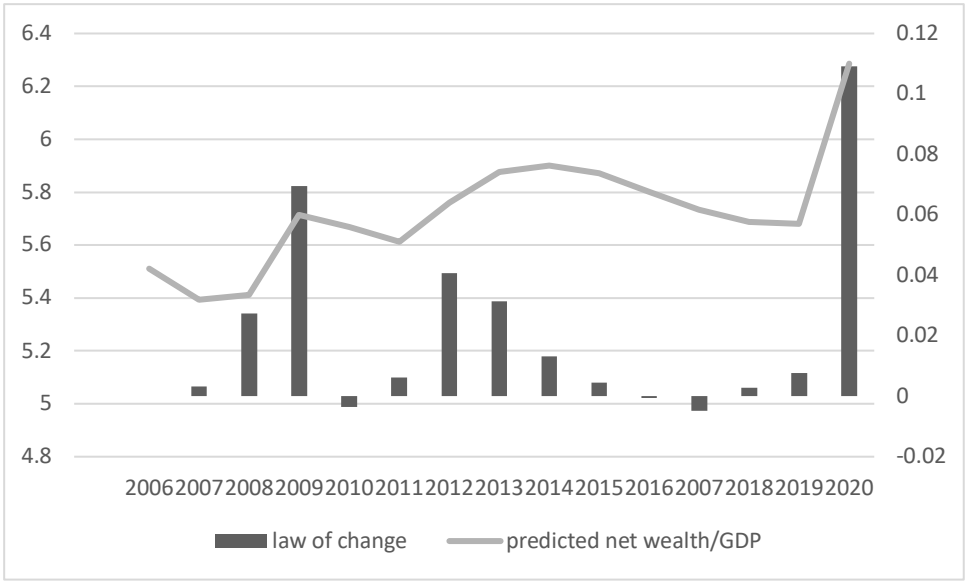

Figure 3 Predicted net (of capital depreciation) wealth to GDP ratio: Italy 2006-2020.

The figures also show the effects of the concomitant increase in the saving rate and decrease in the income growth rate experienced in 2020 because of the Covid pandemic: the wealth to GDP ratio sharply increases.

Even if the period is relatively short, one can see the differences in the

evolution of the debt/GDP and debt/wealth ratios (Figure 4). Between 2015 and 2019, with a decreasing wealth to GDP ratio, the former slightly decreases, while the latter slightly increases; because of the economic consequences of the pandemic, instead, the signal of the worsening in the situation of public finances is stronger with the debt to GDP than with the wealth to GDP indicator.

Even if the differences between the two parameters is not particularly high in the period under observation, the data show that, when the wealth to GDP ratio varies, the debt to GDP ratio might not be a good proxy of the debt to wealth one.

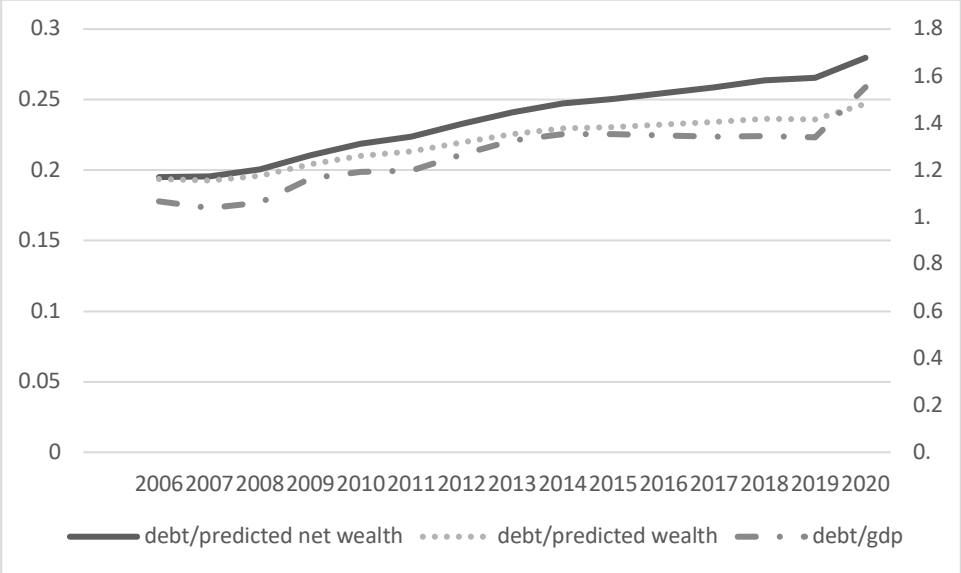

Figure 4 Debt/GDP and debt/wealth ratios: Italy 2006-2020

Based on the argument that the debt to (the demand for) wealth is the proper indicator of the financial situation of the public sector, one should also consider the public debt that is held by a country's central bank and, until it remains there, does not constitute a problem for public finances. As for Italy, these are the total holdings of the Eurosystem, therefore of the Bank of Italy and European Central Bank. However, the European Central Bank is classified among non-resident creditors, therefore it is not possible to enucleate its national debt holdings. Figure 5 shows debt holdings by the Bank of Italy in

the years 2005-2021. One can see that its holdings of government debt have increased, in GDP percentage, to 33,6 and 38,1 in 2020 and 2021, respectively, as a result of the European government bond purchase programs in response to the pandemic. If one considers government debt net of the Bank of Italy's holdings (*debt-* in the Figures), the debt ratios are lower and their evolution follows more similar paths (Figure 6).

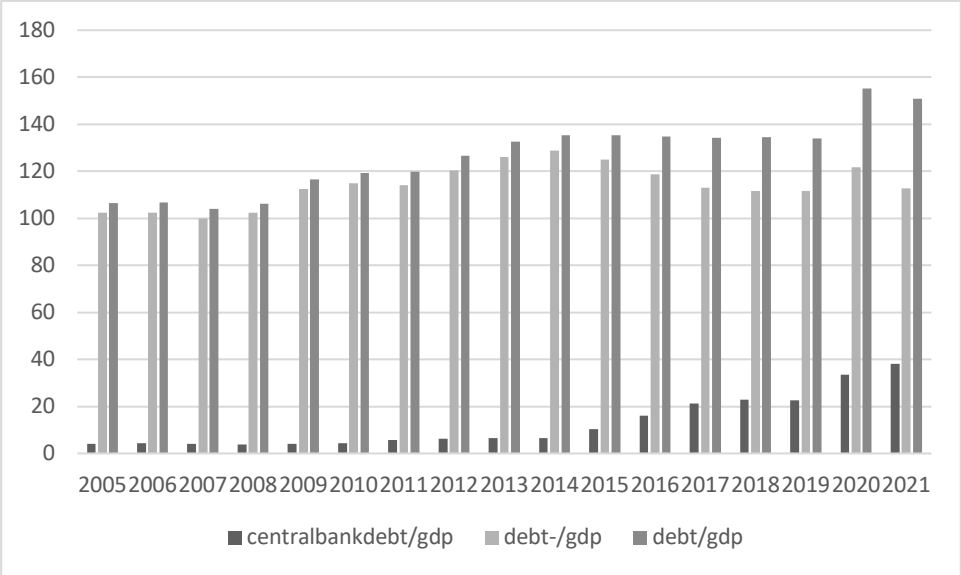

Figure 5 Public debt holdings by the Bank of Italy (2005-2021). Source: European Central Bank

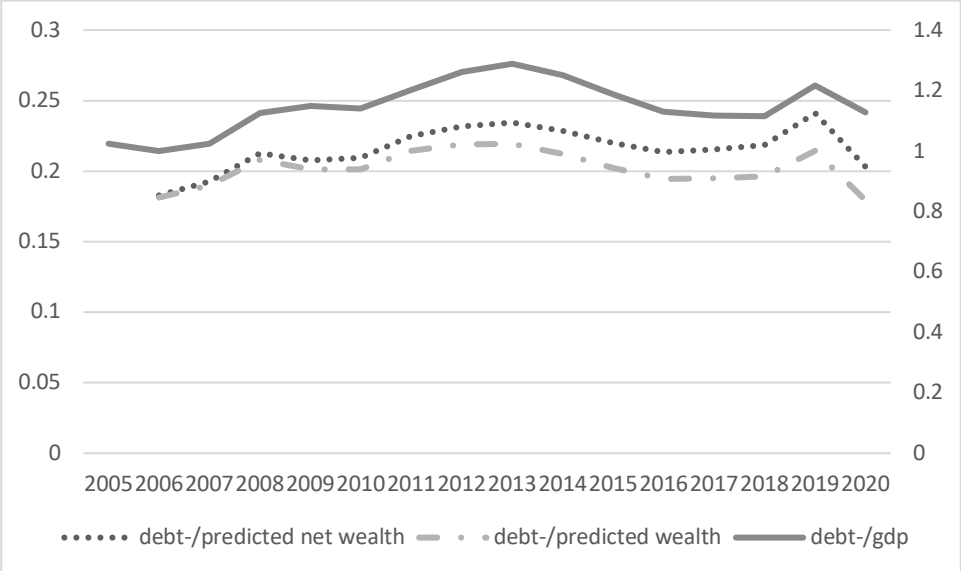

Figure 6 Debt (net of Bank of Italy's holdings)/GDP and debt (net of Bank of Italy's holdings)/wealth ratios: Italy 2006-2020

The fact that the debt to GDP ratio might not be a good proxy of the debt to wealth one suggests that their use as indicators of fiscal stability will yield different results.

To show this, let us consider a modified version of the fiscal sustainability indicator in Blanchard et al (1990):

$$S_n^w = \frac{aspn}{w} - \frac{b_0}{w}(r - g^w), \qquad (4)$$

where *aspn* is the average primary surplus in the next *n* years; w is private wealth; b_0 is government debt in the year preceding the average period; *r* is the real interest rate on government debt, and *g* the growth rate of wealth (*n* year period average). The real interest rate is computed as the difference between the ratio of interest expenditure to debt and the product between the GDP deflator and the initial debt stock. Data on primary surpluses are taken from IMF Fiscal Monitor, April 2022, those on interest expenditure from ECB Statistical Data Warehouse.

Blanchard et al (1990) propose a medium term indicator with n=5 and a long term one with n=40. Because of the unavailability of data (IMF data on primary surpluses are available only until 2027), n=5 and n=9 are used. Predicted net wealth data are used. The results are then compared with those obtaining with the usual GDP normalisation (S_n^y).

$S \geq 0$ indicates that sustainability is achieved over the time span considered. A negative value, instead, indicates that fiscal adjustment is needed.

Figures 7 and 8 illustrate the results for the five-year and nine-year terms, respectively, confirming some difference between the two indicators (the adjustment for the wealth based indicator is also expressed relatively to GDP to eliminate the scale effect).

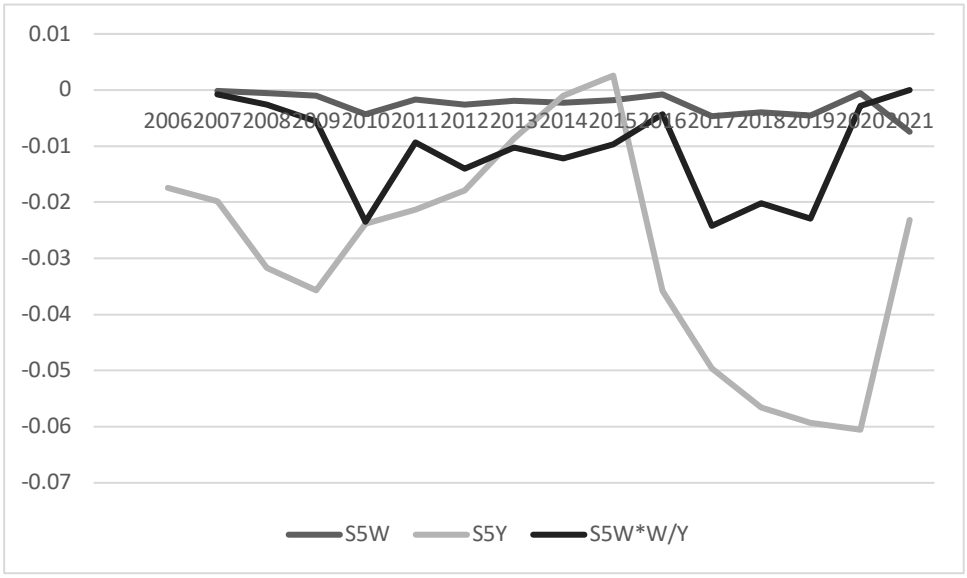

Figure 7 Five-year stability indicator with wealth (S5W) and GDP (S5Y) normalization: Italy 2006-2021

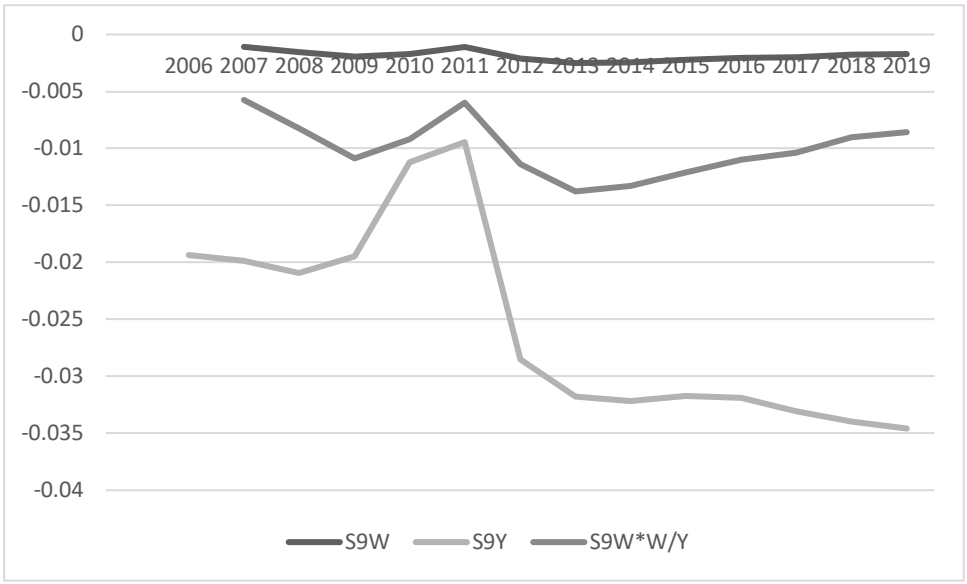

Figure 8 Nine-year stability indicator with wealth (S5W) and GDP (S5Y) normalization: Italy 2006-2019

4 CONCLUSIONS

The debt to GDP has assumed a more relevant role within the EU fiscal rules, especially after the specification of the debt reduction benchmark – so called one-twentieth rule.

Fiscal rules have been suspended in March 2020 to counter the effects of the COVID-19 pandemic on euro-area economies. Even before the pandemic, there was a consensus on the necessity of reforming them. The rigidity of the numeric, uniform Maastricht criteria had already led to more flexible, but also more complex, rules, that, however, produced adverse effects, limiting public investment and the possibility of expansionary fiscal policies after the financial crisis. The pandemic has made it necessary to review the fiscal rules because of the change in the economic scenario with respect to the Maastricht one: public debt has increased (therefore, rules imposing fast adjustments could be counterproductive). The cost of debt is no longer simply connected to its level, given that interest rates are low; the issuing of common debt by means of the Recovery Plan.

Given the uncertainty and the economic effects of the war in Ukraine, the general escape clause will not be activated until 2024. The restoration of the Stability and Growth Pact should be accompanied by a reform of the fiscal rules, aimed at avoiding some of its drawbacks: the pro-cyclical effects of the numerical rules; the contraction in public investment due to budgetary restrictions (the EU average decreased from 3.8 in GDP percentage in 2009 to 2.8 per cent in 2016, and then climbed to 3.3 per cent in 2020; in Italy the ratio went from 3.7 per cent in 2009 to 2.1 per cent in 2018 and rose to 2.6 per cent in 2020, still below the EU average); the non-observability of some of the indicators to be estimated for the application of the rules (output gap and elasticity of the cyclical components of the budget with respect to the tax base, among others); the flexibility and scope for interpretation of the rules.

Among the reform proposals, the debt to GDP ratio should become the anchor of the new framework, together with a revision of its target level and the pace at which it should be achieved (there could be different targets and adjustment paths based on the specific conditions of each individual country). Monitoring should be performed by means of only one indicator, similar to the current

expenditure rule. Compared with the current expenditure rule, the benchmark growth rate would be calculated to allow the target level of the debt anchor to be achieved within the specified adjustment horizon. If the debt/GDP ratio were above the anchor, the nominal growth in net expenditure would have to be below the nominal GDP growth rate, where the latter is calculated on the basis of estimates of actual, potential or trend real GDP growth in future years and an inflation rate assumption. The rate could be the expected rate or one consistent with the objectives of the European Central Bank (ECB). Moreover, the introduction of a "golden rule" should allow the use of deficit funding to finance expenditure with long-term benefits (e.g., investments for the climate transition and digital transformation) or European public goods (research and innovation, defense, security, energy independence, financial stability). A more complex challenge is in assessing proposals that suggest a simplification of the rules, using the debt/GDP ratio only as a medium-term benchmark and net expenditure growth as the sole indicator for annual monitoring.

According to these reform proposals, therefore, the debt to GDP ratio would become the pivot of the new system of rules. The arguments exposed in this contribution suggest cautiousness in its application and hint at its replacement by some wealth based indicator.

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Appendix 1

To derive equations (2) and (3) in the text, consider an open economy version of the Solow growth model as in Milbourne (1997), Benge and Wells (1998) and Gaffney and Rogers (1999). Call GNP X, GDP Y, saving S, investment I, net exports Z, the exogenously given world interest rate r, net foreign assets F, consumption C. We have:

$$Y = C + I + Z \qquad \text{(A1)}$$

$$X = Y + rF = C + S \qquad \text{(A2)}$$

from which we get:

$$C + I + Z + rF = C + S \qquad \text{(A3)}$$

$$S = I + Z + rF \qquad \text{(A4)}$$

which is the fundamental accounting identity for an open economy.

Dynamics enters the model through the growth of the capital stock, K, and foreign assets.

Define gross investment as:

$$I = \frac{dK}{dt} + \delta K \qquad \text{(A5)}$$

where δ is the depreciation rate.

The dynamics of foreign assets is given by:

$$\frac{dF}{dt} = Z + rF \qquad \text{(A6)}$$

The constant returns to scale production function (that is assumed to satisfy the Inada conditions) is:

$$Y(t) = G[K(t), E(t)] \qquad \text{(A7)}$$

where *E* is the number of effective workers, changing because of population growth and technical progress:

$$E = E_0 e^{(n+m)t} \qquad \text{(A8)}$$

where *n* is the population growth rate and *m* the technology growth rate.

Constant returns to scale technology implies that, the intensive from of the production function is:

$$\frac{Y}{E} = G\left(\frac{K}{E}, 1\right) = y = g(k) \qquad \text{(A9)}$$

where $y = \frac{Y}{E}, k = \frac{K}{E}$.

Given that in a small open economy with perfect capital mobility, the interest rate *r* is exogenous, the profit maximising condition $g'(k) = \delta + r$ determines the desired level of capital, $k^* = g'^{-1}(\delta + r)$; with no adjustment costs and perfect capital mobility *y* and *k* adjust immediately to their profit maximising levels, $y^* = g(k^*) = g[g'^{-1}(\delta + r)]; K(t) = g'^{-1}(\delta + r)E(t); Y = G[g'^{-1}(\delta + r), 1]E(t)$. Note that steady state capital per effective worker and GDP per effective

worker depend on world interest rate and depreciation only. In addition, since income *x* can differ from output *y*, the adjustments of *x* and wealth depend on the behaviour of net foreign assets, which in turn depends on the domestic saving ratio.

From (A5) one gets investment per effective worker):

$$i = \frac{dk}{dt} + (m + n + \delta)k \qquad \text{(A10)}$$

which implies at steady state:

$$i^* = (n + m + \delta)k^* \qquad \text{(A11)}$$

Note that, in a closed economy, investment depends on savings; in this case, instead, perfect capital mobility implies that k jumps to k*.

As for the evolution of foreign assets:

$$\frac{dF}{dt} = Z + rF \qquad \text{(A12)}$$

that, in intensive form, becomes:

$$\frac{df}{dt} = z + rf - (n + m)f \qquad \text{(A13).}$$

Assuming that saving is a fixed proportion of current income:

$$\hat{s} = \frac{S}{E} \qquad \text{(A13)}$$

and expressing (A4) in per effective worker terms

$$\hat{s} = i + z + rf \qquad \text{(A14)}$$

$$z = \hat{s} - i - rf \qquad \text{(A15)}$$

and substituting into (A13):

$$\frac{df}{dt} = \hat{s} - i - (n + m)f \qquad \text{(A16)}$$

If saving is a constant fraction of gross current income, the saving rate plays a role in transitional dynamics. This is no longer true if saving is a fixed proportion of permanent income, that is, of the steady state value of wealth: the steady state solutions do not change, but the saving rate no longer plays a role in transitional dynamics.

Putting

$$S = sX \qquad \text{(A17)}$$

$$\hat{s} = sx = s(y + rf) = s(g(k) + rf) \qquad \text{(A18)}$$

and substituting in (16) for $\hat{s}$ and i^*:

$$\frac{df}{dt} = sg(k^*) + srf - nk^* - mk^* - \delta k^* - nf - mf \qquad \text{(A19)}$$

$$\frac{df}{dt} + (n+m-sr)f = sg(k^*) - (n+m+\delta)k^*, \qquad \text{(A20)}$$

which is the differential equation for foreign assets (since k^* is fixed in a small open economy, it is a simple first order differential equation), that provides the steady state value of *f*:

$$f^* = \frac{sg(k^*) - (n+m+\delta)k^*}{n+m-sr} \qquad \text{(A21)}$$

The stability requirement $(n+m-sr)>0$ follows from examining the complete solution, that, for $(n+m-sr)\neq 0$, is:

$$f(t) = [f^* - f(0)]e^{-(n+m-sr)t} + f^* \qquad \text{(A22)}$$

and, for $n+m-sr=0$ is

$$f(t) = f(0) + (sg(k^*) - (n+m+\delta)k^*)t \qquad \text{(A23)}$$

Stability requires $r<\frac{n+m}{s}$ and $s<\frac{n+m}{r}$. This means that any interest rate equal or higher than $\frac{n+m}{s}$ is incompatible with the existence of a steady state (there would be no bound on capital movements); the same applies to a saving ratio higher than $\frac{n+m}{r}$.

Following Gaffney and Rogers (1999), to distinguish between capital importers (net debtors) and capital exporters (net creditors), find the steady state value $f^*=0$:

$$f^* = \frac{sg(k^*) - (n+m+\delta)k^*}{n+m-sr} = 0 \qquad \text{(A24)}$$

which implies

$$s = \bar{s} = \frac{(n+m+\delta)k^*)}{g(k^*)} \qquad \text{(A25)}$$

Substituting into (21):

$$f^* = \frac{(s-\bar{s})g(k^*)}{n+m-sr} \qquad \text{(A26)}$$

and, given that

$$x^* = y^* + rf^* \qquad \text{(A27)}$$

one obtains the steady state value of *x*:

$$x^* = g(k^*) + \frac{r(s-\bar{s})g(k^*)}{n+m-sr} \qquad \text{(A28)}$$

$$x^* = g(k^*)\left[1 + \frac{r(s-\bar{s})}{n+m-sr}\right] \quad \text{(A29)}$$

$$x^* = \left(\frac{n+m-\bar{s}r}{n+m-sr}\right) g(k^*) \quad \text{(A30)}$$

which implies that:

- if $s = \bar{s}$, $f^*=0$ at steady state and $x^* = y^*$, that is, GNP=GDP
- if $s < \bar{s}$, $f^*<0$ at steady state and $x^* = y^*$, that is, the economy is a capital importer (net debtor)
- if $s > \bar{s}$, $f^*>0$ at steady state and $x^* = y^*$, that is, the economy is a capital exporter (net creditor)

The steady state value of consumption per effective worker is:

$$c^* = (1-s)x^* \quad \text{(A31)}$$

$$c^* = (1-s)\left(\frac{n+m-\bar{s}r}{n+m-sr}\right) g(k^*) \quad \text{(A32)}$$

Wealth per effective worker is defined as:

$$w = k + f$$

Note that the steady state value of k is tied down by the exogenous rate of interest. Thus, the dynamics of w and f are simply related. We have:

$$\frac{dw}{dt} + (n+m-sr)(w-k^*) = sg(k^*) - (n+m+\delta)k^* \quad \text{(A33)}$$

$$\frac{dw}{dt} + (n+m-sr)w = s[g(k^*) - rk^*] - \delta k^* \quad \text{(A34)}$$

which yields the steady state value of w:

$$w^* = \frac{s[g(k^*)-rk^*]-\delta k^*}{n+m-sr} \quad \text{(A35)}$$

with the stability condition $(n+m-sr) > 0$.

To obtain the steady state value of z:

$$\frac{df}{dt} + (n+m-r)f = z \quad \text{(A36)}$$

which at steady state reduces to:

$$z^* = (n+m-r)f^* \quad \text{(A37)}$$

The ratio between the steady state value of wealth and of income is given by:

$$\frac{w^*}{x^*} = \frac{s(g(k^*) - rk^*) - \delta k^*}{(n+m-sr)} \frac{(n+m-sr)}{(n+m-\bar{s}r)g(k^*)}$$
$$= \frac{s(g(k^*) - rk^*) - \delta k^*}{(n+m)g(k^*) - (n+m+\delta)\frac{k^*}{g(k^*)}rg(k^*)}$$
$$= \frac{s(g(k^*) - rk^*) - \delta k^*}{(n+m)(g(k^*) - rk^*) - \delta k^* r}$$

which, for $\delta = 0$, yields:

$$\frac{w^*}{x^*} = \frac{s}{(n+m)} \qquad \text{(A38)}$$

which is eq. (2) in the text. Analogously, the ratio between wealth and domestic production is given by:

$$\frac{w^*}{y^*} = \frac{s(g(k^*) - rk^*) - \delta k^*}{(n+m-sr)g(k^*)} = \frac{s(g(k^*) - rk^*) - \delta k^*}{(n+m)g(k^*) - srg(k^*)}$$
$$= \frac{s - sr\frac{k^*}{g(k^*)} - \delta\frac{k^*}{g(k^*)}}{(n+m-sr)}$$

which, for $\delta = 0$, yields

$$\frac{w^*}{y^*} = \frac{s\left(1 - r\frac{k^*}{g(k^*)}\right)}{(n+m-sr)} \qquad \text{(A39)}$$

where $r\frac{k^*}{g(k^*)}$ is the domestic capital share, which is eq. (3) in the text.

Appendix 2

The Italian social security system is largely based on mandatory social security contributions that constitute a form of mandatory savings. To consider their contribution to the evolution of wealth as predicted by the theoretical model of the previous sections, we add them to saving flows. Social security contributions are taken from the Istat database. Figure 7 and 8 illustrate the new debt to wealth ratios.

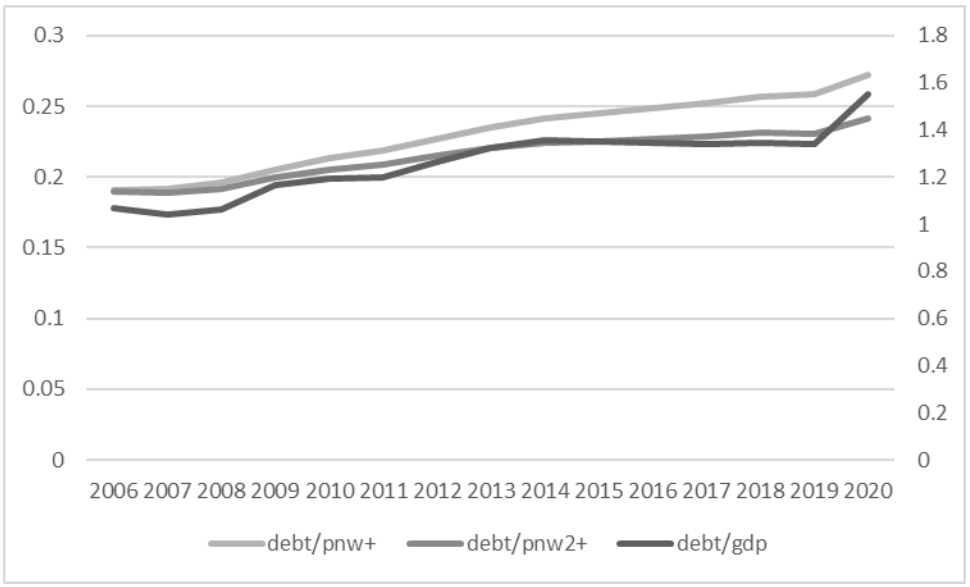

Figure A1 Debt/GDP and debt/wealth ratios (predicted wealth and net wealth including mandatory contributions saving flows – pnw2+ and pnw+, respectively): Italy 2006-2020

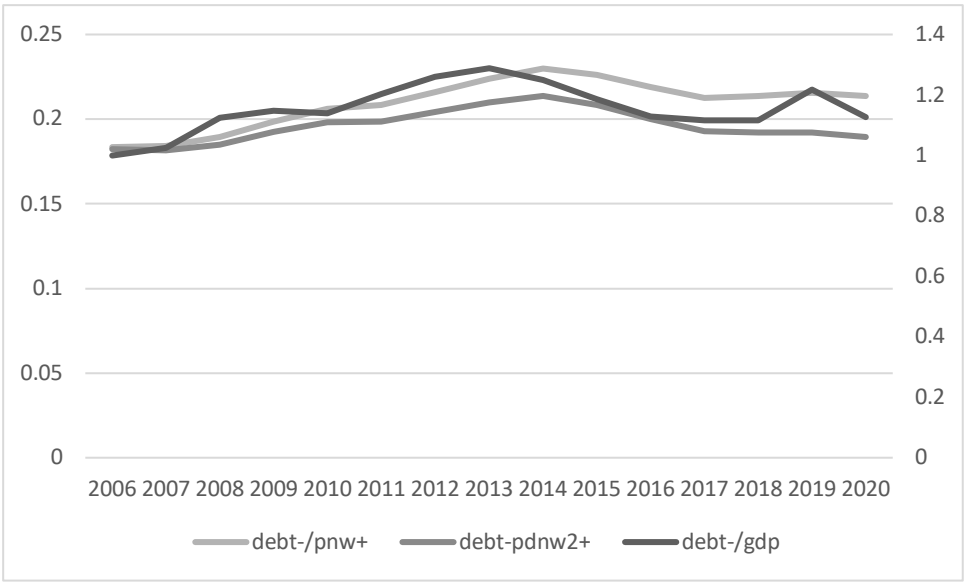

Figure A2 Debt (net of Bank of Italy's holdings)/GDP and debt (net of Bank of Italy's holdings)/wealth ratios (predicted wealth and net wealth including mandatory contributions saving flows – pnw2+ and pnw+, respectively): Italy 2006-2020.

https://www.dsge.uniroma1.it/pubblicazioni/istituto-economia-e-finanza/public-finance-research-papers