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**AN ANALYSIS ON OPTIMAL TAXATION AND ON POLICY CHANGES IN
AN ENDOGENOUS GROWTH MODEL WITH PUBLIC EXPENDITURE**

THOMAS RENSTRÖM AND LUCA SPATARO

Thomas Renström, University of Durham (UK).
Email: t.i.Renström@durham.ac.uk.

Luca Spataro, Dipartimento di Economia e Management, University of Pisa (Italy).
Email: l.spataro@ec.unipi.it.

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Abstract

In this work we analyse the issue of optimal taxation and of policy changes in an endogenous growth model driven by public expenditure, in the presence of endogenous fertility and labour supply. While normative analysis confirms the Chamley-Judd result of zero capital income tax, positive analysis reveals that the presence of endogenous fertility produces different results as for the effects of taxes on total employment.

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1. Introduction

An extensive literature on optimal taxation of factor incomes in a general equilibrium-dynamic framework has been flourishing in the last three decades. A well-established finding of such works is that, in the long run, capital income should not be taxed, thus shifting the burden from factor income taxation toward labor (Judd, 1985, Chamley, 1986, Judd, 1999). Although the result is robust with respect to several extensions, some exceptions may arise, such as in the case of borrowing constraints (Aiyagari 1995 and Chamley 2001), market imperfections (Judd 1997), incomplete set of taxes (Correia 1996, Cremer et al. 2003), overlapping generations (Eros and Gervais 2002), social discounting and disconnected economies (De Bonis and Spataro 2005, 2010), government time-inconsistency and lack of commitment (Reis 2012), externalities from suboptimal policy rules (Turnovsky 1996).

The case of externalities is particularly relevant for endogenous growth models: Romer (1986) introduces externalities deriving from existing capital (spillovers as “learning by doing”); Lucas (1988) shows that decreasing returns to capital could be avoided by adopting a broad view of capital itself that entails human capital as well (externalities from “human capital”); in Barro (1990), spillovers from productive public expenditure avoid diminishing returns to capital and are the engine of sustained long run growth; finally, in a subsequent work, Romer (1990) himself applies the concept of nonrivalry to “ideas” or “discoveries” that can enhance production efficiency and technological progress, and obtained increasing returns in production and thus sustained per-capita income growth.¹

In this work we extend the analysis of optimal taxation and public expenditure policies to an endogenous growth setting with productive public expenditure by allowing for endogenous labour supply and endogenous fertility. This has been never done so far.

In fact, several works have analysed the impact of fiscal policies on economic growth, such as Barro (1990), Jones and Manuelli (1990) and Rebelo (1991). As for welfare analysis, Lucas (1990) and Turnovsky (1992) compare the effects of a tax on capital versus a tax on labour and find the former to be inferior to the latter from the viewpoint of economic welfare. Turnovsky (1996) analyses the issue of first-best optimal taxation and expenditure policies in an endogenous growth model with externalities stemming from public goods both in the utility and in the production function and Turnovsky (2000) extends the analysis to the case of endogenous labour supply².

In this type of models, direct taxation brings about a natural trade-off: on the one hand, it distorts incentives to save and work, hence reducing growth; on the other, it increases the marginal productivity of private inputs, thus increasing growth and possibly welfare. This is the key contribution of Barro (1990), which was extended in several subsequent studies.

¹ The literature on endogenous technological change through R&D activities, schumpeterian competition and spillovers has been evolving over the last decade (for a review see Acemoglu 2009 chapters 13 and subsequent ones).

² Discussions of the effects of taxation in models of endogenous labour supply are also provided by Rebelo (1991).

However, in all these works population growth is either absent or exogenous. In fact, the observed large variations in fertility rates both across countries and across times, has led an increasing number of scholars to work on the reformulation of economic theory of endogenous fertility and on the provision of different social criteria for allocation efficiency with variable population.³ Moreover, most of the endogenous growth models mentioned above suffer from the “scale effect”, meaning that the steady-state growth rate increases with the size (scale) of the economy, as indexed, for example, by population. In order to overcome this problem non-scale models have been provided by Jones (1995) and subsequent works, although still hinging on exogenous population.

In order to breach this gap, in the present work we extend Barro (1990) model, in which the engine of growth is productive public expenditure, by allowing for both endogenous labour supply (as in Turnovsky 2000) and endogenous population (as in Spataro and Renstrom 2012) and we use this model to analyse fiscal policy in the form of distortionary taxes and government expenditure changes.

We retain Barro (1990) approach since there is consolidated evidence that public expenditure in favour of productive services have a sizable impact on growth (for major insights, see, among others, Turnovsky 1996, García Peñalosa and Turnovsky, 2005). More precisely, we carry out our analysis under two different types of public expenditure: a) optimal amount of public services (as in Barro 1990); b) fixed fraction of GDP (either fixed at the optimal level or not – as in Turnovsky 1996).

We also note that our model allows to avoid two shortcomings of the aforementioned nonscale growth models: first, the direct positive link between economic and demographic growth entailed, which is not supported by post-war data (see Agemoglu 2009, p.)⁴ and, second, the fact that the long-run equilibrium growth rate is determined by technological parameters and is independent of macro policy instruments.

The assumption of endogenous population, however, poses major issues related to welfare analysis. In fact, given that under these circumstances welfare evaluations typically imply the comparisons between states of the world in which the size of population is different, the Pareto criterion cannot be used. To overcome this issue, we adopt individual preferences that allow for social orderings that are based on desirable welfarist axioms in presence of variable population.

We show that, while some well-established results on second-best taxation extend also to our model, positive analysis produces results that are somehow in contrast with the existing literature, due to the presence of endogenous fertility.

The work is organized as follows: in section 2, after laying out the expenditure flow model, with both suboptimal and optimal government expenditure, we characterize the optimal taxation rules and public debt; in section 3 we perform the

³ See, among others, Becker and Barro (1988), Barro and Becker (1989), Jaeger and Kuhle (2009), Yew and Zhang (2009), de la Croix et al. (2012), Golosov et al. (2007), Renstrom and Spataro (2011).

⁴ See Renstrom and Spataro (2014) for an endogenous growth model driven by human capital with variable population, where the relationship between population growth and economic growth is not necessarily positive.

same in the public capital model. In section 4 we present a tax reforms analysis, in order to verify the impact of the latter on economic growth. Section 5 concludes.

2. The model

In this section we lay out the benchmark model. We denote individual quantities by lower case letters, and aggregate quantities by corresponding upper case letters, so that $V = Nv$, with N population size. As anticipated above, we find it instructive to present both the case in which public expenditure is chosen optimally (model 1) and or is fixed as a fraction of GDP (model 2). In the latter case, we will first assume that the share of public expenditure is set arbitrarily while in the second stage it is set optimally. This approach will enhance our understanding of the optimal capital income tax rate which, as it will be shown, depends upon both the socially optimal level of government expenditure and the deviation of actual expenditure from its social optimum.

2.1. Households

We assume that the representative agent is endowed with a unit of time that can be allocated either to leisure or the work or to child rearing. We also assume, for the sake of simplicity, that each generation lives for one period, and life-time utility is $u(c_t, l_t)$, where c_t is life-time consumption for that individual, l_t is labour supply. We assume that utility is increasing in c_t , decreasing in l_t and strictly concave. We also follow the convention that $u(0, \cdot) = 0$ represents neutrality at individual level (i.e. if $u < 0$ the individual prefers not to have been born), and denote the critical level utility as α .

An individual family chooses consumption, labour supply, savings and the number of children (i.e. the change in the cohort size N).

We also assume that raising children is costly. We nest the existing approaches in the literature by assuming the cost per family member in the number of children, $\theta(n)$, can either be linear (as in Becker and Barro, 1989, Cremer et al. 2006) or strictly convex (as in Tertilt 2005 and Growiec 2006). Convex cost implies decreasing returns to scale in child rearing.

As for firms, we assume perfectly competitive markets and constant return to scale technology. The consequence of the assumptions on the production side is that we retain the “standard” second-best framework, in the sense that there are no profits and the competitive equilibrium is Pareto efficient in absence of taxation. Otherwise there would be corrective elements of taxation. Finally, we assume the government finances an exogenous stream of per-capita expenditure x , that enters as an input in private sector production function, by issuing debt and levying taxes.

To retain the second-best, we levy taxes on the choices made by the families, i.e. savings, labour supply and children. Consequently we introduce the capital-income tax and labour income tax, possibly conditional on the number of children.

2.1.1 Preferences

We focus on a single dynasty (household) or a policymaker choosing consumption and population growth over time, so as to maximize:

$$W(u_{t-1}, N_t, u_{t+1}, N_{t+1}, \dots) = \sum_{s=0}^{\infty} \beta^s N_{t+s} [u_{t+s} - \alpha u_{t-1+s}] \quad (1)$$

where N_t is the population (family) size of generation t , $u_t = u(c_t, l_t) \geq 0$ is the instantaneous utility function of an individual of generation t , such that $u(0, \cdot) = 0$, $u_c > 0, u_l < 0, u_{cc} < 0, u_{ll} > 0$, $\beta \in (0, 1)$ the intergenerational discount factor and αu_{t-1} is the Critical Level Utility, with $\alpha \in (0, 1)$ applied to generation t .

The critical level is defined as a utility value (αu_{t-1} in our case) of an extra person that, if added to the (unaffected) population, would make society as well off as without that person (see a la Blackorby et al. 1995). Critical Level Utilitarianism (CLU) allow for axiomatically founded social preferences and avoid the so-called Repugnant Conclusion ((RC henceforth; see Parfit 1976, 1984, Blackorby et al. 2002)⁵, which, in a growth model with endogenous population, takes the form of upper-corner solution for the population rate of growth (society reproduces at its physical maximum rate).⁶

However, we depart from classical CLU by assuming that the critical value is a positive function of previous generation's utility (only if αu_{t-1} is a constant this social ordering would coincide with the CLU). Renström and Spataro (2012, 2015) refer to this population criterion as "Relative CLU" (RCLU). RCLU, is in the spirit of the Critical Level Utilitarianism (that is axiomatically founded) but under these preferences the judgment (the critical level of utility for life worth living) is relative to the existing generation's level of wellbeing. In other words, a society or a household at low level of utility will set a lower threshold of utility for the next generation, and a society with high living standard will set a higher level. So if parents had a good life, they require their children to have a good life as well, and vice versa.

Notice that such preferences can be also derived by aggregating over individuals entailed with altruistic preferences on future generations and relative utility on previous generation's⁷, so that the current analysis can be interpreted as being either normative or positive.

More precisely, we will assume the following form of the intratemporal utility function:

$$u(c, l) = \frac{c^{1-\sigma}}{1-\sigma} \Psi(1-l) \quad (2)$$

with $\Psi' > 0$, $\Psi'' < 0$, which is the assumption to work in presence of sustained long-run per-capita income growth. The continuous time version of (1) can be written as (see Appendix A):

⁵ According to the RC, any state in which each member of the population enjoys a life above neutrality is declared inferior to a state in which each member of a larger population lives a life with lower utility (Blackorby et al. 1995, 2002).

⁶ See Renström and Spataro (2011) for a discussion and Spataro and Renström (2012) for an optimal taxation and policy change analysis.

⁷ See Renström and Spataro (2015).

$$U = \int_0^{\infty} e^{-\rho t} N_t u(c_t, l_t) [1 - \alpha(n_t - \bar{\rho})] dt. \quad (3)$$

Moreover, in continuous time the population dynamics is described by the following law:

$$\frac{\dot{N}_t}{N_t} = n_t \quad (4)$$

with $n_t \in [\underline{n}, \bar{n}]$. The integral is finite only if $(\rho - \bar{n}) > 0$, which we assume throughout the paper.

Hence, the problem of the household is to maximize (3) under the constraint:

$$\dot{A}_t = \bar{r}_t A_t + \bar{w}_t (l_t - \theta(n_t)) s_t(n_t) N_t - c_t N_t \quad (5)$$

where A_t is household wealth, $\bar{r}_t = r_t(1 - \tau_t^k)$ is net-of-tax interest rate, $\bar{w}_t s_t(n_t) = w_t(1 - \tau_t^l) s_t(n_t)$ is the net-of-tax wage and τ_t^k, τ_t^l are the tax rate on capital income, on labour income, respectively and $s_t(n_t)$ is a tax/subsidy function on labour income that depends on the number of children. We assume that child rearing cost $\theta(n_t)$ is a time cost and is specified over the number of children each parent has, so that it is a function of the population growth rate. In fact, in equilibrium each parent has the same number of children, so the per family member population growth rate becomes the economy wide one. We assume it is increasing in number of children and either linear or convex, $\theta' > 0, \theta'' \geq 0$.⁸

Furthermore, we assume that there are lower and upper bounds on the population growth rate: $n_t \in [\underline{n}, \bar{n}]$. Realistically, there is a physical constraint at each period of time on how many children a parent can have. There is also a constraint on how low the population growth can be. The reason for the latter assumption is twofold: first, we do not allow individuals to be eliminated from the population (in that there is no axiomatic foundation for that); moreover, even if nobody wants to reproduce there will always be accidental births. Clearly, from eq. (1) the problem has a finite solution only if $\rho > \bar{n}$ which we assume throughout our analysis.

⁸ Notice that convex childrearing costs, although questionable in terms of realism, are commonly used in population literature (see, among others Tertilt 2005, Growiec 2006), in that convexity is necessary for avoiding a corner solution for n . In our work, constant or linear costs for raising children would still insure an interior solution for n , provided that α is nonzero. In other words, if α is assumed to be zero, then Classical Utilitarianism would apply but convex costs would be required for avoiding a corner solution for n .

2.2. Firms

We assume constant-returns-to-scale production technology with labour-augmenting productive public expenditure. More precisely, the production function is:

$$F_t = F(K_t, x_t L_t) = F(K_t, x_t N_t (l_t - \theta(n_t))) = TK_t^\gamma (x_t N_t (l_t - \theta(n_t)))^{1-\gamma} \quad (6)$$

with T the parameter representing total factor productivity, K_t is capital stock, assumed infinitely durable, x_t the labour-augmenting flow of services from government spending on the economy's infrastructure. We also assume that these services are not subject to congestion so that x_t is a pure public good. $L_t = (l_t - \theta(n_t))N_t$ is hired labour, with $l_t - \theta(n_t)$ the fraction of time dedicated to work.

Assuming perfect competition, firms hire capital, K , and labour services, L , on the spot market and remunerate them according to their marginal productivity, such that

$$F_{K_t} = r_t \quad (7)$$

$$F_{L_t} = w_t \quad (8)$$

Moreover, the economy resource constraint is:

$$\dot{K}_t = F(K_t, x_t L_t) - c_t N_t - x_t N_t. \quad (9)$$

2.3. The government

We allow the government to finance an exogenous stream of public expenditure x_t by levying taxes, both on capital and labour income and issuing debt, B , whose law of motion is:

$$\dot{B}_t = r_t B_t - \tau_t^k r_t A_t - \tau_t^l (l_t - \theta(n_t)) s_t(n_t) w_t N_t + \tau_t^l (l_t - \theta(n_t)) [s_t(n_t) - 1] w_t N_t + x_t N_t. \quad (10)$$

The expenditure flow model that we present in this section will be allowed to take two possible forms:

- 1) Expenditure model 1: $X = xN$
- 2) Expenditure model 2: $xN = \delta F$

In words, in the first model the public policy consists in a constant per capita flow of services x , while in the latter public expenditure is a fixed fraction of total output.

Finally, we can summarize the economy's resource constraint as follows:

$$\dot{K}_t = \tilde{F}_t - c_t N_t, \quad (9')$$

where

$$\tilde{F} = F - xN \text{ in model 1} \quad (11)$$

and

$$\tilde{F} = F - xN = (1 - \delta) \delta^{\frac{1-\gamma}{\gamma}} T^{\frac{1}{\gamma}} (l - \theta(n))^{\frac{1-\gamma}{\gamma}} K \text{ in model 2}^9 \quad (12)$$

3. The decentralized equilibrium

We now characterize the decentralized equilibrium of the economy. The problem of the individual (household) is to maximize (3) subject to (5), taking A_0 and N_0 as given. The current value Hamiltonian is¹⁰:

$$H_t = N_t u_t G_t + q_t [\bar{r}_t A_t + \bar{w}_t (l_t - \theta(n_t)) s_t(n_t) N_t - c_t N_t] + \lambda_t n_t N_t \quad (13)$$

with $G_t = [1 - \alpha(n_t - \rho)] > 0$, q_t and λ_t the shadow price of wealth and of population, respectively.

The first-order conditions are the following¹¹:

$$\frac{\partial H}{\partial A} = \rho q - \dot{q} \Rightarrow \dot{q} = (\rho - \bar{r})q \quad (14)$$

$$\frac{\partial H}{\partial c} = 0 \Rightarrow G u_c = q \quad (15)$$

$$\frac{\partial H}{\partial l} = 0 \Rightarrow -G u_l = q \bar{w} s(n) \quad (16)$$

$$\frac{\partial H}{\partial N} = \rho \lambda - \dot{\lambda} \Rightarrow \dot{\lambda} = (\rho - n)\lambda - uG - q[\bar{w}(l - \theta(n))s(n) - c] \quad (17)$$

⁹ In fact, by using $xN = \delta F = \delta T K^{\gamma} (xN(l - \theta(n)))^{1-\gamma}$ we get that $xN = (\delta T)^{\frac{1}{\gamma}} ((l - \theta(n)))^{\frac{1-\gamma}{\gamma}} K$, such that $F = \delta^{\frac{1-\gamma}{\gamma}} T^{\frac{1}{\gamma}} (l - \theta(n))^{\frac{1-\gamma}{\gamma}} K$ and $\tilde{F} = F - xN = (1 - \delta) \delta^{\frac{1-\gamma}{\gamma}} T^{\frac{1}{\gamma}} (l - \theta(n))^{\frac{1-\gamma}{\gamma}} K$.

¹⁰ We focus on interior solutions for n , so that the potential constraint $n_t \in [\underline{n}, \bar{n}]$ is not binding.

¹¹ We omit the subscript referring to time when it causes no ambiguity to the reader.

$$\frac{\partial H}{\partial n} = 0 \Rightarrow \alpha u = \lambda - q\bar{w}[s(n)\theta'(n) - s'(n)(l - \theta(n))] \quad (18)$$

and the transversality conditions are

$$\lim_{t \rightarrow \infty} e^{-\rho t} q_t A_t = 0, \quad \lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t N_t = 0 \quad (19)$$

The last condition for the competitive equilibrium is capital market clearing condition:

$$A_t = K_t + B_t$$

which, in per capita terms, is:

$$a_t = k_t + b_t. \quad (20)$$

Note that, under policy 2, equilibrium market price (interest rate) is equal to (private) marginal product of capital, and the latter can be different from social marginal product of capital: in fact, we get that:

$$r = F_K = \gamma T \left(\frac{xN}{K} \right)^{1-\gamma} (l - \theta(n))^{1-\gamma} = \gamma \delta^{\frac{1-\gamma}{\gamma}} T^{\frac{1}{\gamma}} (l - \theta(n))^{\frac{1-\gamma}{\gamma}} \quad (21)$$

while the social marginal product of capital is:

$$r^* = \tilde{F}_K = (1 - \delta) \delta^{\frac{1-\gamma}{\gamma}} T^{\frac{1}{\gamma}} (l - \theta(n))^{\frac{1-\gamma}{\gamma}} \quad (22)$$

This difference is due to the presence of externality brought about by public expenditure. More precisely, we get that

$$r \begin{matrix} > \\ < \end{matrix} r^* \Leftrightarrow \gamma \begin{matrix} > \\ < \end{matrix} (1 - \delta) \quad (23)$$

In case the policymaker aims to correct for this externality, it can either choose δ optimally (i.e. equal to the production elasticity of public expenditure, $1 - \gamma$, as in Turnovsky 1996, for example), or raise corrective taxes.

Balanced growth path

Finally, we characterize the balanced growth path (BGP), along which all per-capita variables grow at the same rate.

By using eq. (14), doing time derivative of (15) and recognizing that, along the BGP n and l are constant, we get the usual expression for the per-capita consumption growth rate, g :

$$g \equiv \frac{\dot{c}}{c} = \frac{\bar{r} - \rho}{\sigma} \quad (24)$$

We can notice that, as usual, the economy growth rate is proportional to the net-of-tax interest rate. This, in turn, depends on the whole set of the endogenous variables, namely, the population growth rate, labour supply, and capital intensity, and on the deep parameters of the economy, comprising taxes. We will address the effects of the latter on the economy growth rate in section 5, after characterizing the optimal taxation rules.

4. The Ramsey problem

We now solve the optimal tax problem (Ramsey problem). In doing so, we adopt the primal approach, consisting of the maximization of a direct social welfare function through the choice of quantities (i.e. allocations; see Atkinson and Stiglitz 1972)¹². For this purpose we must restrict the set of allocations among which the government can choose to those that can be decentralized as a competitive equilibrium. We now provide the constraints that must be imposed on the government's problem in order to comply with this requirement.

In our framework there is an implementability constraint associated with the individual family's intertemporal consumption choice. More precisely this constraint is the individual budget constraint with prices substituted for by using the consumption Euler equation, which yields (see Appendix A.2):

$$G_0 A_0 u_{c_0} = \int_0^{\infty} e^{-\rho t} G_t [u_{c_t} c_t + u_{l_t} (l_t - \theta(n_t))] N_t dt \quad (25)$$

Finally there three feasibility constraints, one which requires that private and public consumption plus investment be equal to aggregate output (eq. 9); the other is given by eq. (17) (again, where we make use of individuals's FOCs (15) and (16)), the last one is eq. (4).

Hence, supposing that the policy is introduced in period 0, the problem of the policymaker is to maximize (1) subject to eq. (25), and, $\forall t \geq 0$, eqs. (9'), (17) and (4). Hence, the current value Hamiltonian is:

$$H_t = N_t G_t [u_t + \mu (u_{c_t} c_t + u_{l_t} (l_t - \theta(n_t))) + \eta_t [\tilde{F}_t - c_t N_t] + \phi_t n_t N_t \quad (26)$$

which can be written as (omitting time subscripts):

$$H = NG(1 + \mu)u + N\mu G[u_c c + u_l (l - \theta(n)) - u] + \eta [\tilde{F} - cN] + \phi nN \quad (27)$$

where μ , η and ϕ are the multipliers associated with the constraints.

¹² On the contrary, the dual approach takes prices and tax rates as control variables. For a survey see Renström (1999).

First order condition with respect to consumption is:

$$\frac{\partial H}{\partial c} = NG(1 + \mu)u_c + (N\mu + \omega)G\Delta_c u_c - \eta N = 0 \quad (28)$$

with $\Delta_c \equiv \frac{u_{cc}c + u_{cl}(l - \theta(n))}{u_c}$ and $\Delta_l \equiv \frac{u_{cl}c + u_{ll}(l - \theta(n))}{u_l}$ usually referred to as the “general equilibrium elasticity” of consumption and of labour, respectively.

By using eq. (15), (28) can be written as:

$$\frac{\partial H}{\partial c} = 0 \Rightarrow 1 + \mu(1 + \Delta_c) = \frac{\eta}{q} \quad (29)$$

As for labour supply, FOC yields:

$$\frac{\partial H}{\partial l} = NG(1 + \mu)u_l + N\mu G\Delta_l u_l + \eta w^* N = 0 \quad (30)$$

with $w^* \equiv \frac{1}{N} \frac{\partial \tilde{F}}{\partial l}$; hence, by using eq. (16) we get:

$$\frac{\partial H}{\partial l} = 0 \Rightarrow 1 + \mu(1 + \Delta_l) = \frac{\eta w^*}{q\bar{w}s(n)} \quad (31)$$

Finally,

$$\frac{dH}{dK} = \eta r^* = \rho\eta - \dot{\eta} \Rightarrow \dot{\eta} = (\rho - r^*)\eta \quad (32)$$

with $r^* \equiv \frac{1}{K} \frac{\partial \tilde{F}}{\partial K}$

$$\frac{dH}{dN} = G(1 + \mu)u + \mu G[u_c c + u_l(l - \theta(n)) - u] - \eta c + \phi n = \rho\phi - \dot{\phi} \quad (33)$$

$$\frac{dH}{dn} = -\alpha N \{(1 + \mu)u + \mu[u_c c + u_l(l - \theta(n)) - u]\} - N\mu G u_l \theta'(n) - \eta w^* N \theta'(n) + \phi N = 0 \quad (34)$$

We can provide the following Proposition for the case when x is chosen optimally:

Proposition 1. In model 1, along the optimal BGP, second best optimal taxation implies:

$$\tau^k = 0, \text{ and } (1 - \tau^l)s(n) = \frac{1 + \mu(1 + \Delta_c)}{1 + \mu(1 + \Delta_l)} \in (0,1)$$

Proof. See Appendix .B1 □

We now characterize the second best tax structure under the assumption that public expenditure is set as a fixed fraction of GDP.

Proposition 2. In model 2, along the optimal BGP, second best optimal capital tax

$$\tau^k \begin{matrix} > \\ < \end{matrix} 0 \text{ iff } \delta \begin{matrix} > \\ < \end{matrix} 1 - \gamma. \text{ Sufficient for nonnegative effective labour income tax is}$$

$$\delta \begin{matrix} > \\ < \end{matrix} 1 - \gamma$$

Proof. See Appendix B.2. □

The analysis of second best optimal taxation carried out so far shows that the Chamley-Judd result holds also in our scenario, provided that labour income taxation can be conditioned on the number of children that are present in the household: in fact, along the BGP, capital income tax should be zero and effective labour income tax should be positive. Notice that without the $s(n)$ function the second best policy would not be implementable by the policymaker. Moreover, as in Turnovsky (1996), nonzero capital income tax arises, although in a second best context, for correcting suboptimal public expenditure.

In fact, when the fraction of public expenditure is above (below) the social second-best optimum, the social return to capital is less than its private marginal physical product. Consequently, capital income should be taxed to obtain the social optimum.

Since the novelty of our paper is the possibility to condition labour income taxation on the number of children, we now characterise the shape of $s(n)$ along the BGP.

Proposition 3: In models 1 and 2, along the BGP, for any $s(n)$ function assigned to the household, the following holds at the second best optimum:

$$\frac{s'(n)}{s(n)} = -\frac{1}{z} - Q < 0 \tag{35}$$

$$\text{with } z \equiv \rho - n - \frac{(1 - \sigma)}{\sigma} (\bar{r} - \rho) > 0 \text{ and } Q = \mu \frac{\Delta_l - \Delta_c - 1}{1 + \mu(1 + \Delta_c)} \frac{\theta'(n)}{l - \theta(n)}$$

Proof. See Appendix B,3 □

The proposition above states that the component of labour income taxes that is dependent on the number of children should be a decreasing function. The reason is that, when the policymaker raises taxes on labour income, this will increase the

number of children (see also Proposition 6). Hence, the $s(n)$ function will be shaped in such a way to counterbalance this increase.

Finally, it is possible to provide a shape of the $s(n)$ function that is implementable by the policymaker.

Proposition 4: An $s(n)$ function implementing the second best is:

$$s(n) = \pi(\varepsilon)(\Pi - n)^\varepsilon. \quad (36)$$

Where $\pi(\varepsilon) > 0$, $\Pi > n$ and $0 < \varepsilon \leq 1$ are policy instruments taken as given by the household.

Proof: See Appendix B.4. □

While π , Π and ε are taken as given by the household, in the second best equilibrium they are functions of equilibrium quantities.

Finally, we can characterize the same function in the case of fixed costs for raising children.

Corollary 1: If $\theta'(n)$ is equal to zero, the function $s(n)$ implementing the second best is $s(n) = \Pi - n$, where $\Pi = \rho - \frac{(1-\sigma)}{\sigma}(\bar{r} - \rho)$ and \bar{r} are taken as given by the household.

Proof: When $\theta'(n) = 0$ Q is equal to zero and it is clear from (35) that $s=z$ implements the second best equilibrium. Eq. (B.8) when $Q=0$, substituted into (36) gives $z = \Omega(\varepsilon)(\varepsilon z)^\varepsilon$, yielding $z^{1-\varepsilon} = \Omega(\varepsilon)\varepsilon^\varepsilon$, which holds at $\varepsilon=1$ and $\Omega(1) = 1$. □

Note that in this case there are fewer extra policy instruments needed, in that $s(n)$ is simply a function of n and Π , the latter being set directly by the policymaker at its (second best) equilibrium value.

Finally, the Proposition that follows provides the result concerning the sign of the optimal level of debt:

Proposition 5: Under model 1 and 2 optimal debt is negative.

Proof. See Appendix B.5. □

This result states that along the second-best optimal BGP public expenditure should entirely be financed by labour income taxes and by the returns of public assets (in model 1) or by also capital income taxes (in model 2, if $\gamma > (1 - \delta)$).

5. Tax reforms

We now analyse the effects of policy changes on the equilibrium levels of the main variables (i.e. labour supply $l - \theta(n)$, population growth rate n and the growth rate

of the economy, g , which is proportional to \bar{r} , in that $dg = d\bar{r} / \sigma$, along the BGP. To simplify the analysis and without loss of generality, we assume $s(n)=1$ and that childbearing costs are linear, so that $\theta''(n) = 0$ ¹³. The analysis below encompasses both models 1 and 2.

From decentralized equilibrium, we get (see Appendix C.1):

$$\frac{\alpha}{G} \frac{1}{1-\sigma} = \frac{\frac{\sigma}{1-\sigma} + \frac{\bar{w}}{c} (l-\theta(n))}{\bar{r}-n-\frac{\bar{r}-\rho}{\sigma}} - \frac{\theta'(n)}{l-\theta(n)} \frac{\bar{w}}{c} (l-\theta(n)) \quad (37)$$

$$\frac{\Psi'(l-\theta(n))}{\Psi} \frac{1}{1-\sigma} = \frac{1-\gamma}{\gamma} \frac{(1-\tau^l)r}{r^*-n-\frac{\bar{r}-\rho}{\sigma}}. \quad (38)$$

and

$$\bar{r} = (1-\tau^k)F_K = (1-\tau^k)\gamma\delta^{\frac{1-\gamma}{\gamma}} T^{\frac{1}{\gamma}} (l-\theta(n))^{\frac{1-\gamma}{\gamma}} \quad (39)$$

With the economy always being on a balanced growth path, the effects of government policy on the equilibrium are obtained by taking the differentials of (37)-(132). Routine calculations yield the qualitative responses with respect to τ^k and τ^l which we can summarize in the following proposition:

Proposition 6. Along the BGP, the effects of policy changes are the following:

$$\frac{\partial g}{\partial \tau^l} < 0, \frac{\partial(l-\theta(n))}{\partial \tau^l} < 0, \frac{\partial n}{\partial \tau^l} > 0$$

$$\frac{\partial g}{\partial \tau^k} < 0, \frac{\partial(l-\theta(n))}{\partial \tau^k} > 0, \frac{\partial n}{\partial \tau^k} < 0$$

Proof. See Appendix C.1. □

The results on economic growth are somehow intuitive: higher taxes produce lower growth. However, differently from previous literature on endogenous growth models and endogenous labour supply, the effects of taxes on employment may be different. For example, in Turnovsky 2000 the signs of the effect of either taxes on labour supply are quite the same and negative. The difference in our results are due to the presence of endogenous fertility. In other words, it emerges that the transmission channel of fiscal policies on economic growth are qualitatively different: on the one hand, the labour income tax, while depressing wages, reduces the incentive for net accumulation but reduces the opportunity cost of raising children (i.e. net-wage; see eq. 16). This will increase the number of children per household and reduce the household's time dedicated to work.

¹³ See footnote 8.

On the other hand, the capital income tax, while reducing the incentives towards net accumulation (lower net interest rates) as well, increases the incentives of devoting more time on the job (and relatively less to raising children); finally, by hitting future consumption, a higher capital income tax exerts a reinforcing effect towards the reduction of the number of descendents.

Finally, given that the effects of taxes on the BGP equilibrium variables are qualitatively different, one might wonder what are the effects of policies that redistribute the burden of taxation between production inputs.

For this purpose we focus on the case in which the government operates a redistribution of taxes in such a way that total tax revenues (TR) remain fixed proportion of GDP. Needless to say, we will only consider those cases in which the economy is on the “upward-sloped trait” of the Laffer curve, meaning that any change of either tax rate will imply a change of the opposite sign in the other tax,

i.e. $\left. \frac{d\tau^l}{d\tau^k} \right|_{(TR/Y)} < 0$. Otherwise, there would be room for decreasing both taxes while

keeping the b/k ratio constant.

Given that we have shown that the changes in the capital income and labour income taxes have opposite effect on labour supply and population growth, respectively, the results of the constant-debt-redistributive policy on the latter variables are clear. However, in principle the effects on the economy growth rate are ambiguous: for example, an increase of capital income will tend to reduce g , while the corresponding decrease of the labour income tax will exert a positive effect on it. The final effect on the BGP rate of growth will depend on which force dominates. The results of this exercise are summarized in the following proposition:

Proposition 7. Along the BGP, a tax reform consisting in an increase (decrease) of the capital income tax and a corresponding reduction (increase) of the tax on labour income in such a way to leave the total-tax-revenues/GDP ratio unchanged, implies that equilibrium labour supply increases (decreases) and the population growth rate decreases (increases), while the change of the growth rate of per-capita income is ambiguous. However, sufficient for the latter to be negative (positive) is that the capital income tax is not lower than the labour income tax.

Proof. See Appendix C.2.

The content of Proposition 5 has clear-cut policy implications: since the effects of the redistributive policy are dominated by those of capital income taxes, if a policymaker aims at boosting per-capita growth, with constant tax revenues over GDP, it should redistribute the burden of taxation towards labour income and maintain the labour income tax lower than the capital income tax. This, in turn, would also increase the rate of growth of population. The latter result can be explained in terms of “quality-quantity” trade-off (higher taxes on labour income reduce the opportunity cost of raising children) already unveiled by several works on population economics.

7. Conclusions

In the present work we have carried out an analysis of optimal taxation and policy changes in an endogenous growth model in presence of endogenous fertility and labour supply. As far as the normative analysis is concerned, we show that, at the steady state the second-best policy entails zero capital income tax, positive labour income tax and negative debt. Optimal nonzero tax on capital income results as a corrective device only in the case of suboptimal public spending, as in Turnovsky (1996), although in a second best analysis.

From a positive standpoint we show that a rise of taxes (either on labour income or on capital income), depresses per-capita growth. However, while an increase of fiscal pressure on labour input reduces labour supply and increases the population growth rate, an increase of taxes on capital produces the opposite results. This result is in contrast with existing literature (see for example Turnovsky 2000), due to the presence of endogenous fertility.

Finally, we have also analysed the effects of a fiscal policy aiming at redistributing the tax burden in such a way to maintain tax revenues a fixed proportion of GDP. The analysis has shown that the effects are qualitatively the same of the case of a capital tax change, although the sign of the change of the BGP rate of growth is in general ambiguous. Sufficient for such a variation to be unambiguous is that the capital income tax is greater than or equal to the labour income tax.

Consequently, the latter result suggests that an economy that wishes to boost economic growth without resorting to extra public debt, should reduce capital income taxes *and* increase labour income taxes, while keeping both taxes roughly of the same magnitude. This policy, in turn, would also increase the population rate of growth.

In this paper we have treated public expenditure as a flow variable (services from current expenditure). A natural extension of our study is to analyse the case of public expenditure as financing a *stock* of public goods (infrastructure): this case is left for future research.

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Appendix

Appendix A.1: The form of eq. (2) (drawn from Renström and Spataro 2012)

By starting from eq. (1) and collecting utility terms of the same date, the welfare function W can be written as:

$$W = \sum_{t=0}^{\infty} \beta^t N_t u(c_t) [1 - \alpha \beta (1 + n_t)] - \alpha N_0 u(c_{-1}) \quad (\text{A.1})$$

By ignoring c_{-1} as it is irrelevant for the planning horizon, and defining $\beta = \frac{1}{1 + \rho}$

we get:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1 + \rho} \right)^t N_t u(c_t) \left(1 - \alpha \frac{1 + n_t}{1 + \rho} \right). \quad \text{In continuous time, by approximating}$$

$$\frac{1 + n_t}{1 + \rho} \approx -(\rho - n_t) \text{ the latter expression can be written as follows:}$$

$$U = \int_0^{\infty} e^{-\rho t} N_t u(c_t) [1 - \alpha(n_t - \rho)] dt \quad \square \quad (\text{A.2})$$

Appendix A.2. Implementability constraint

First, let us take the following time derivative:

$$\frac{d}{dt}(q_t A_t) = \dot{q}_t A_t + q_t \dot{A}_t$$

which, exploiting eqs. (5) and (14) can be written as

$$\frac{d}{dt}(q_t A_t) = (\rho - \bar{r}_t) q_t A_t + q_t [\bar{r}_t A_t + \bar{w}_t (l_t - \theta(n_t)) s_t(n_t) N_t - c_t N_t]$$

Using (15) and (16) it follows:

$$\frac{d}{dt}(q_t A_t) - \rho q_t A_t = -N_t G_t [u_c c_t + u_l (l_t - \theta(n_t))]$$

Hence, pre-multiplying by $e^{-\rho t}$ and integrating both sides, making use of transversality conditions and of eq. (15), we obtain eq. (25) in the text:

$$G_0 A_0 u_{c_0} = \int_0^{\infty} e^{-\rho t} G_t [u_c c_t + u_l (l_t - \theta(n_t))] N_t dt$$

Appendix B.1. Proof of proposition 1

As for the capital income tax, from (32) we get:

$$\dot{\eta} = (\rho - r^*) \eta$$

and given that $\frac{\eta}{q}$ must be constant along the BGP (by eq. 22), the rate of growth of η

and q must be equal, so that $\bar{r} = r^*$.

Given that, in model 1, $r = r^*$, then $\tau^k = 0$.

As for the labour income tax, from (29) and (31)

$$\frac{\bar{w} s(n)}{w^*} = \frac{1 + \mu(1 + \Delta_c)}{1 + \mu(1 + \Delta_l)} \quad (\text{B.1})$$

Given that in model 1 $w = w^*$, then

$$(1 - \tau^l) s(n) = \frac{1 + \mu(1 + \Delta_c)}{1 + \mu(1 + \Delta_l)}$$

with

$$\Delta_c = -\sigma - \frac{\Psi'}{\Psi}(l - \theta) < 0$$

$$\Delta_l = 1 - \sigma - \frac{\Psi''}{\Psi'}(l - \theta) > 0. \quad \square$$

Appendix B.2. Proof of Proposition 2

As for capital income tax, given that, it must be true that $\bar{r} = r^*$ and in model 2, $r = \frac{\gamma}{(1-\delta)}r^*$, it follows that $\tau^k = 1 - \frac{1-\delta}{\gamma} > 0$ iff $\delta > 1 - \gamma$.

As for effective labour income tax, given that $w^* = \frac{(1-\gamma)}{\gamma}(1-\delta)\delta^{\frac{1-\gamma}{\gamma}}T^{\frac{1}{\gamma}}\frac{K}{N}(l-\theta(n))^{\frac{1-2\gamma}{\gamma}}$ and $w = w^*\frac{\gamma}{1-\delta}$, eq. (B.1) implies:

$$(1-\tau^l)s(n) = \left(\frac{1-\delta}{\gamma}\right) \left[\frac{1+\mu(1+\Delta_c)}{1+\mu(1+\Delta_l)} \right]$$

which can be greater or smaller than 1. Given $\Delta_l > 0$ and $\Delta_c < 0$, if $\gamma - (1-\delta) > 0$, then effective labour income taxation is positive (i.e. $(1-\tau^l)s(n) < 1$). \square

Appendix B.3. Proof of Proposition 3

Preliminarily, note that, by eq. 2 and the definition of Δ_c :

$$\frac{u_c c + u_l(l - \theta(n)) - u}{u} = \Delta_c \quad (\text{B.2})$$

so that, eq. (34), when divided by u becomes:

$$\frac{\dot{\phi}}{u} = \alpha[1 + \mu + \mu\Delta_c] - \mu G \frac{\Psi'}{\Psi} \theta'(n) + \frac{\eta}{q} G \frac{\Psi'}{\Psi} \frac{w^*}{\bar{w}s(n)} \theta'(n). \quad (\text{B.3})$$

where we have used $\frac{u_l}{u} = -\frac{\Psi'}{\Psi}$ and $\frac{\eta}{u} w^* = \frac{\eta}{q} \frac{q\bar{w}s(n)}{u} \frac{w^*}{\bar{w}s(n)} = \frac{\eta}{q} G \frac{\Psi'}{\Psi} \frac{w^*}{\bar{w}s(n)}$.

Along the BGP, $\frac{\Psi'}{\Psi}$ and $\Delta_c = -\sigma - \frac{\Psi'}{\Psi}(l - \theta(n))$ are constant and $\frac{\eta}{q} G \frac{\Psi'}{\Psi} \frac{w^*}{\bar{w}s(n)}$

is constant as well, so that $\frac{\dot{\phi}}{u}$ is constant. (33) gives

$\frac{\dot{\phi}}{u} = (\rho - n) \frac{\phi}{u} - G(1 + \mu + \mu\Delta_c) + \frac{\eta c}{u}$; using the latter and eq. (24) we have

$$\frac{d}{dt} \left(\frac{\phi}{u} \right) = \frac{\dot{\phi}}{u} - \frac{\phi \dot{u}}{u^2} = z \frac{\phi}{u} - G(1 + \mu + \mu\Delta_c) + \frac{\eta c}{u} = 0, \quad \text{which yields}$$

$$z \frac{\phi}{u} = G(1 + \mu + \mu\Delta_c) - \frac{\eta c}{u}. \text{ Exploiting the latter, eq. (B.3) gives:}$$

$$\left[-\alpha + \frac{G}{z} \right] (1 + \mu + \mu\Delta_c) - \frac{\eta c}{zu} = G \frac{\Psi'}{\Psi} [1 + \mu\Delta_l] \theta'(n) \quad (\text{B.4})$$

Notice that $\frac{\eta c}{zu} = \frac{\eta q c}{zqu} = \frac{\eta G u_c c}{zqu} = \frac{\eta G}{q z} (1 - \sigma)$. Next, (29), (B.4) becomes

$$\left[-\alpha + \sigma \frac{G}{z} \right] (1 + \mu + \mu\Delta_c) = G \frac{\Psi'}{\Psi} [1 + \mu\Delta_l] \theta'(n) \quad (\text{B.5})$$

Now, dividing eq. (18) by u gives

$$\alpha = \frac{\lambda}{u} - \frac{q\bar{w}}{u} [s(n)\theta'(n) - s'(n)(l - \theta(n))] \quad (\text{B.6})$$

As noticed above, $\frac{q\bar{w}}{u}$ is constant along the BGP and so is $\frac{\lambda}{u}$.

Hence:

$$\frac{d}{dt} \left(\frac{\lambda}{u} \right) = \frac{\dot{\lambda}}{u} - \frac{\lambda \dot{u}}{u^2} = z \frac{\lambda}{u} + G\Delta_c = 0 \quad (\text{B.7})$$

where we used (17) and (B.2), so that $z \frac{\lambda}{u} = -G\Delta_c$. Combining the latter with (B.6) we have:

$$\alpha = -\frac{G}{z} \Delta_c - \frac{q\bar{w}}{u} [s(n)\theta'(n) - s'(n)(l - \theta(n))]$$

Exploiting $\Delta_c = -\sigma - \frac{\Psi'}{\Psi} (l - \theta)$, (16) and (2) we obtain

$$\alpha - \sigma \frac{G}{z} = \frac{G}{z} \frac{\Psi'}{\Psi} (l - \theta(n)) - G \frac{\Psi'}{\Psi} \left[\theta'(n) - \frac{s'(n)}{s(n)} (l - \theta(n)) \right]$$

Finally, exploiting (B.5) we obtain (35) □

Appendix B.4. Proof of Proposition 4

Differentiating (36) with respect to n and combining with (35) we obtain:

$$\varepsilon = (\Pi - n) \left(\frac{1}{z} + Q \right) \quad (\text{B.8})$$

Any pair of ε and Π satisfying (B.8) potentially implements the second best equilibrium. The only caveat is that first order condition of the household with respect to n must characterize a maximum and not a minimum; therefore the derivative of (28) with respect to n should be non-positive, i.e.

$$-\theta''(n)s(n) - 2\theta'(n)s'(n) + (l - \theta(n))s''(n) \leq 0$$

To incorporate the case of linear costs, where $\theta''(n) = 0$, sufficient for the equation above to hold, for our $s(n)$ function (36) is $2\theta'(n)(\Pi - n) + (l - \theta(n))(\varepsilon - 1) \leq 0$. Using (B.8) the latter inequality implies:

$$\frac{1 - \varepsilon}{\varepsilon} \geq 2 \frac{\theta'(n)}{(l - \theta(n))} \frac{z}{1 + zQ} \quad (\text{B.9})$$

It is clear that (B.9) can be satisfied as LHS $\rightarrow +\infty$ when $\varepsilon \rightarrow 0$, so that LHS can be made arbitrarily large. For any ε satisfying (B.9), the corresponding Π is found by eq. (B.8). \square

Appendix B.5. Proof of Proposition 5

Let us start from inputs remuneration:

$$\frac{\partial \tilde{F}}{\partial l} = \left(\frac{1 - \gamma}{\gamma} \right) (1 - \delta) \delta^{\frac{1-\gamma}{\gamma}} T^{\frac{1}{\gamma}} (l - \theta(n))^{\frac{1-2\gamma}{\gamma}} K. \quad \text{By exploiting the definition of}$$

$$w^* \equiv \frac{1}{N} \frac{\partial \tilde{F}}{\partial l} \quad \text{then} \quad w^*(l - \theta(n)) = \left(\frac{1 - \gamma}{\gamma} \right) \frac{\tilde{F}}{N}.$$

$$\text{Next, } \frac{\partial \tilde{F}}{\partial K} = \left(\frac{1 - \gamma}{\gamma} \right) (1 - \delta) \delta^{\frac{1-\gamma}{\gamma}} T^{\frac{1}{\gamma}} (l - \theta(n))^{\frac{1-\gamma}{\gamma}} \equiv r^* = \frac{\tilde{F}}{K}$$

$$\text{So, } \frac{w^*(l - \theta(n))}{r^* k} = \left(\frac{1 - \gamma}{\gamma} \right) \frac{\tilde{F}}{N} \frac{K}{\tilde{F}} \frac{1}{k} = \left(\frac{1 - \gamma}{\gamma} \right)$$

Next rewriting $\dot{K}_t = \tilde{F}_t - c_t N_t$ in per capita terms, we get:

$$\frac{\dot{k}}{k} = \frac{\tilde{F}}{k} - \frac{c}{k} - n = r^* - \frac{c}{k} - n, \quad \text{so that}$$

$$\frac{c}{k} = r^* - n - \frac{\dot{k}}{k}. \quad \text{Given that along the BGP } \frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \frac{r^* - \rho}{\sigma} \quad \text{we get that:}$$

$$\frac{c}{k} = r^* - n - \frac{r^* - \rho}{\sigma} \equiv z$$

Next, let us exploit the individuals' budget constraint,

$$\dot{a} = (\bar{r} - n)a + \bar{w}(l - \theta(n))s(n) - c. \quad (\text{B.10})$$

and given that

$$\frac{\dot{a}}{a} = \frac{\dot{c}}{c} = \frac{r^* - \rho}{\sigma} \quad (\text{B.11})$$

it follows:

$$\frac{c}{a} = (\bar{r} - n) - \frac{r^* - \rho}{\sigma} + \frac{\bar{w}}{a}(l - \theta(n))s(n)$$

Moreover, exploiting

$$\frac{\bar{w}(l - \theta(n))s(n)}{a} = \frac{\bar{w}}{w^*} \frac{k}{a} \frac{w^*(l - \theta(n))s(n)}{r^* k} r^* = \frac{\bar{w}s(n)}{w^*} \frac{k}{a} r^* \left(\frac{1 - \gamma}{\gamma} \right)$$

we can rewrite eq. above as:

$$\frac{c}{a} = z + \frac{\bar{w}s(n)}{w^*} \frac{k}{a} r^* \left(\frac{1 - \gamma}{\gamma} \right).$$

Finally,

$$\frac{a}{k} = \frac{\frac{c}{a}}{\frac{z}{a} + \frac{\bar{w}s(n)}{w^*} \frac{k}{a} r^* \left(\frac{1 - \gamma}{\gamma} \right)} = \frac{a}{k} \frac{z}{z \frac{a}{k} + \frac{\bar{w}s(n)}{w^*} r^* \left(\frac{1 - \gamma}{\gamma} \right)}$$

and collecting terms we get:

$$z \left(\frac{a}{k} - 1 \right) + \frac{\bar{w}s(n)}{w^*} r^* \left(\frac{1 - \gamma}{\gamma} \right) = 0$$

and, by recalling that $\left(\frac{a}{k} - 1 \right) = \frac{b}{k}$ we get:

$$\frac{b}{k} = -\frac{\frac{\bar{w}s(n)}{w^*} r^* \left(\frac{1-\gamma}{\gamma} \right)}{\left(r^* - n - \frac{r^* - \rho}{\sigma} \right)} < 0.$$

Appendix C.1. Proof of Proposition 6

Equating eq. (24) and $\frac{\dot{k}}{k} = r^* - n - \frac{c}{k}$ it follows:

$$\frac{c}{k} = r^* - n - \frac{\bar{r} - \rho}{\sigma} \quad (\text{C.1})$$

By eq. (18) and (2)

$$\frac{\alpha u}{qc} = \frac{\alpha}{G} \frac{1}{1-\sigma} = \frac{\lambda}{qc} - \frac{\bar{w}}{c} \theta'(n) \quad (\text{C.2})$$

whereby it follows that along the BGP, $\frac{\lambda}{qc}$ is constant, so that:

$$\frac{\dot{\lambda}}{\lambda} - \frac{\dot{q}}{q} - \frac{\dot{c}}{c} = 0 \quad (\text{C.3})$$

and, using (14), (17) and (24), eq. (C.3) yields:

$$\frac{\lambda}{qc} = \frac{\frac{\sigma}{1-\sigma} + \frac{\bar{w}}{c} (l - \theta(n))}{\bar{r} - n - \frac{\bar{r} - \rho}{\sigma}} \quad (\text{C.4})$$

(which implies that $\bar{r} - n - \frac{\bar{r} - \rho}{\sigma} > 0$).

it follows that, along the BGP path,
Using (18) and (C.4) we get:

$$\frac{\alpha}{G} \frac{1}{1-\sigma} = \frac{\frac{\sigma}{1-\sigma} + \frac{\bar{w}}{c} (l - \theta(n))}{\bar{r} - n - \frac{\bar{r} - \rho}{\sigma}} - \frac{\theta'(n)}{l - \theta(n)} \frac{\bar{w}}{c} (l - \theta(n)) \quad (\text{C.5})$$

Next, by exploiting (15), (16) and (2) one gets:

$$\frac{\Psi'(l - \theta(n))}{\Psi} \frac{1}{1-\sigma} = \frac{\bar{w}}{c} (l - \theta(n)) \quad (\text{C.6})$$

Substituting (C.6) into (C.5) yields:

$$\left(\frac{\alpha}{G}\right)\left(\bar{r}-n-\frac{\bar{r}-\rho}{\sigma}\right)=\sigma+\frac{\Psi'}{\Psi}\left[l-\theta(n)-\theta'(n)\left(\bar{r}-n-\frac{\bar{r}-\rho}{\sigma}\right)\right] \quad (\text{C.7})$$

Finally, using eq. (C.1) and (C.6) we obtain:

$$\frac{\Psi'(l-\theta(n))}{\Psi(1-\sigma)}=\frac{1-\gamma}{\gamma}\frac{(1-\tau^l)r}{r^*-n-\frac{\bar{r}-\rho}{\sigma}}. \quad (\text{C.8})$$

Total differentiation of (C.7),(C.8) and (21), under linear costs for raising children, yields the following system:

$$\begin{cases} \left[-\frac{\Psi''}{\Psi'}+\frac{\Psi'}{\Psi}-\frac{M}{l-\theta(n)}\right]dl+\left[\frac{M}{l-\theta(n)}\theta'(n)-\frac{1}{r^*-\bar{r}+\bar{z}}\right]dn=-\frac{d\tau^l}{1-\tau^l}-\frac{r}{\sigma}\frac{d\tau^k}{r^*-\bar{r}+\bar{z}} \\ -\omega dl-\Omega dn-\frac{1-\sigma}{\sigma}\left(\frac{\alpha}{G}+\frac{\Psi'}{\Psi}\theta'(n)\right)d\bar{r}=0 \\ -\bar{r}\left(\frac{1-\gamma}{\gamma}\right)\frac{dl}{l-\theta(n)}+\bar{r}\left(\frac{1-\gamma}{\gamma}\right)-\frac{\theta'(n)}{l-\theta(n)}dn+d\bar{r}=-rd\tau^k \end{cases}$$

$$\text{with } \Omega \equiv \alpha \frac{1+\alpha\left(\frac{1-\sigma}{\sigma}\right)\left(\bar{r}-n-\frac{\bar{r}-\rho}{\sigma}\right)}{G^2} = \frac{\alpha}{G}\left(1-\bar{z}\frac{\alpha}{G}\right)$$

$$\omega \equiv \frac{\Psi'}{\Psi} + \left[-\frac{\Psi''}{\Psi} + \left(\frac{\Psi'}{\Psi}\right)^2\right] \left[l-\theta(n)-\theta'(n)\bar{z}\right], \quad \bar{z} \equiv \left(\bar{r}-n-\frac{\bar{r}-\rho}{\sigma}\right)$$

$$M \equiv \left[\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{\rho-n}{r^*-\bar{r}+\bar{z}}\right)-1\right]$$

Matrix notation of the system yields:

$$\begin{bmatrix} -\frac{\Psi''}{\Psi'}+\frac{\Psi'}{\Psi}-\frac{M}{l-\theta(n)} & \frac{M}{l-\theta(n)}\theta'(n)-\frac{1}{r^*-\bar{r}+\bar{z}} & 0 \\ -\omega & -\Omega & -\frac{1-\sigma}{\sigma}\left(\frac{\alpha}{G}+\frac{\Psi'}{\Psi}\theta'(n)\right) \\ -\bar{r}\left(\frac{1-\gamma}{\gamma}\right)\frac{1}{l-\theta(n)} & \bar{r}\left(\frac{1-\gamma}{\gamma}\right)-\frac{\theta'(n)}{l-\theta(n)} & 1 \end{bmatrix} \begin{bmatrix} dl \\ dn \\ d\bar{r} \end{bmatrix} = \begin{bmatrix} -\frac{d\tau^l}{1-\tau^l}-\frac{r}{\sigma}\frac{d\tau^k}{r^*-\bar{r}+\bar{z}} \\ 0 \\ rd\tau^k \end{bmatrix} \quad (\text{C.9})$$

Notice that $\Omega > 0$. As for the sign of ω sufficient for $\omega > 0$ is that $[l - \theta(n) - \theta'(n)\bar{z}] > 0$; preliminarily note that, given $\bar{r} > \rho$, it follows:

$$\bar{z} = \left(1 - \frac{1}{\sigma}\right)\bar{r} - n + \frac{\rho}{\sigma} < \left(1 - \frac{1}{\sigma}\right)\rho - n + \frac{\rho}{\sigma} = \rho - n.$$

Now, suppose that $\theta(n) = \bar{\theta} \times (A + n)$ (when $n=0$ population is constant, which means that each adult gives birth to one child, whose cost is $\bar{\theta} \times A$) then $\theta'(n) = \bar{\theta}$. So, it follows that

$$[l - \theta(n) - \theta'(n)\bar{z}] > l - \bar{\theta}(A + n) - \bar{\theta}(\rho - n) = l - \bar{\theta}(A + \rho). \quad (\text{C.10})$$

Assuming that the RHS of eq. (C.10) is positive, it follows that $\omega > 0$. The latter assumption states that, at the equilibrium, the parameters of the economy are such that the time devoted to work after raising the maximum number of children ($n = \rho$) is still positive. We will maintain such hypothesis in the remainder of the proof.

Next, using Cramer's rule, we can obtain the partial derivatives

$$D \frac{\partial n}{\partial \tau^l} = \frac{1}{1 - \tau^l} \left[-\omega - \frac{1 - \gamma}{\gamma} \frac{\bar{r}}{l - \theta} \frac{1 - \sigma}{\sigma} \left(\frac{\alpha}{G} + \frac{\Psi'}{\Psi} \theta' \right) \right] < 0 \quad (\text{C.11})$$

$$D \frac{\partial (l - \theta(n))}{\partial \tau^l} = \frac{1}{1 - \tau^l} (\Omega + \theta' \omega) > 0 \quad (\text{C.12})$$

$$D \frac{\partial \bar{r}}{\partial \tau^l} = \frac{1}{1 - \tau^l} \frac{1 - \gamma}{\gamma} \frac{\bar{r}}{l - \theta(n)} (\Omega + \theta' \omega) > 0 \quad (\text{C.13})$$

where D is the determinant of the matrix at the RHS of system (C.9).

We now show that the sign of D is negative. Since the expression for D is very complicated, in this proof we will exploit the constraint on total revenues resorting to an argument à la Laffer.

Let us recall the expression for total revenues (TR) per unit of capital (this is the variable that is constant along the BGP):

$$tr \equiv \frac{TR}{K} = \left[\tau^l (l - \theta(n)) \frac{w}{k} + \tau^k r \right] = \tau^l r (l - \theta(n)) \frac{w}{rk} + \tau^k r = \frac{\tau^l \frac{1 - \gamma}{\gamma} + \tau^k}{1 - \tau^k} \bar{r}$$

Total differentiation with respect to τ^l yields:

$$\frac{\partial tr}{\partial \tau^l} = \frac{1 - \gamma}{\gamma} \frac{1}{1 - \tau^k} \bar{r} + \frac{\tau^l \frac{1 - \gamma}{\gamma} + \tau^k}{1 - \tau^k} \frac{\partial \bar{r}}{\partial \tau^l}$$

We will focus on economies in which $\frac{\partial tr}{\partial \tau^l} > 0$, which implies that:

$$\frac{\partial \bar{r}}{\partial \tau^l} > -\frac{\frac{1-\gamma}{\gamma} \bar{r}}{\tau^l \frac{1-\gamma}{\gamma} + \tau^k} \quad (\text{C.14})$$

Finally, exploiting eq. (C.13), (C.14) becomes:

$$\frac{\frac{1}{1-\tau^l} \frac{1-\gamma}{\gamma} \frac{\bar{r}}{l-\theta(n)} (\Omega + \theta' \omega)}{D} > -\frac{\frac{1-\gamma}{\gamma} \bar{r}}{\tau^l \frac{1-\gamma}{\gamma} + \tau^k} \quad (\text{C.15})$$

Given that the numerator of the LHS of eq. (C.15) is positive, if D is positive then the inequality would always hold true, for any level of taxes. However, this is a contradiction, because, for $\tau^k = 0$ and $\tau^l \rightarrow 1$ labour supply would shrink to zero and, consequently, total revenues would be zero. Hence, an economic meaningful BGP implies $D > 0$.

In the light of this finding, from eqs. (C.11)-(C.13) we have:

$$\frac{\partial g}{\partial \tau^l} < 0, \frac{\partial(l-\theta(n))}{\partial \tau^l} < 0, \frac{\partial n}{\partial \tau^l} > 0.$$

As for the effects of capital income tax, we get.

$$D \frac{\partial \bar{r}}{\partial \tau^k} = r \frac{(\Omega + \theta' \omega)}{l-\theta(n)} \left(\frac{\bar{r}}{r^* - \bar{r} + \bar{z}} \frac{1-\gamma}{\gamma \sigma} - M \right) + r \Omega \left(-\frac{\Psi''}{\Psi'} + \frac{\Psi'}{\Psi} \right) + \frac{r \omega}{r^* - \bar{r} + \bar{z}} \quad (\text{C.16})$$

As for eq. (C.16), given that

$$\frac{\bar{r}}{r^* - \bar{r} + \bar{z}} \frac{1-\gamma}{\gamma \sigma} - M = \frac{1-\gamma}{\gamma} \frac{\frac{\bar{r}-\rho}{r^* - \bar{r} + \bar{z}} + n}{r^* - \bar{r} + \bar{z}} + 1 > 0 \quad (\text{C.17})$$

it follows that

$$\frac{\partial g}{\partial \tau^k} < 0$$

As for labour supply, it is convenient to compute

$$\begin{aligned}
D \frac{\partial l}{\partial \tau^k} &= -\left(\frac{1-\sigma}{\sigma}\right) \left(\frac{\alpha}{G} + \frac{\Psi'}{\Psi} \theta'\right) \left(\frac{r}{r^* - \bar{r} + \bar{z}}\right) + \frac{r}{\sigma} \frac{\Omega}{r^* - \bar{r} + \bar{z}} + \\
&+ \left(\frac{1-\sigma}{\sigma}\right) \left(\frac{\alpha}{G} + \frac{\Psi'}{\Psi} \theta'\right) \frac{r \theta'}{l - \theta} \left(M - \frac{\bar{r}}{r^* - \bar{r} + \bar{z}} \frac{1-\gamma}{\gamma \sigma}\right)
\end{aligned}
\tag{C.18}$$

by exploiting the definitions of M and Ω eq. (C.18) becomes:

$$\frac{r}{\sigma} \frac{1}{r^* - \bar{r} + \bar{z}} \left[\frac{\alpha}{G} \left(\sigma - \bar{z} \frac{\alpha}{G}\right) - (1-\sigma) \frac{\Psi'}{\Psi} \theta' \right] - \left(\frac{1-\sigma}{\sigma}\right) \left(\frac{\alpha}{G} + \frac{\Psi'}{\Psi} \theta'\right) \left(\frac{r \theta'}{r^* - \bar{r} + \bar{z}}\right) \frac{1}{l - \theta} \left[r^* - \bar{r} + \bar{z} + \frac{1-\gamma}{\gamma} \left(\frac{\bar{r} - \rho}{\sigma} + n\right) \right]
\tag{C.18}$$

Given that by eq. (C.7) $\frac{\alpha}{G} \left(\sigma - \bar{z} \frac{\alpha}{G}\right) = -\frac{\Psi'}{\Psi} (l - \theta - \theta' \bar{z}) < 0$ it follows that

$$D \frac{\partial l}{\partial \tau^k} < 0 \Rightarrow \frac{\partial l}{\partial \tau^k} > 0$$

As for $\frac{\partial n}{\partial \tau^k}$, Cramer's rule provides:

$$D \frac{\partial n}{\partial \tau^k} = \left(\frac{1-\sigma}{\sigma}\right) \left(\frac{\alpha}{G} + \frac{\Psi'}{\Psi} \theta'\right) \left[r \left(-\frac{\Psi''}{\Psi'} + \frac{\Psi'}{\Psi}\right) + \frac{r}{l - \theta} \left(\frac{1-\gamma}{\gamma \sigma} \frac{\bar{r}}{r^* - \bar{r} + \bar{z}} - M\right) \right] + \frac{r}{\sigma} \frac{\omega}{r^* - \bar{r} + \bar{z}}
\tag{C.19}$$

and exploiting (C.17)

$$D \frac{\partial n}{\partial \tau^k} > 0 \Rightarrow \frac{\partial n}{\partial \tau^k} < 0$$

Consequently, as for labour supply,

$$\frac{\partial(l - \theta(n))}{\partial \tau^k} = \frac{\partial l}{\partial \tau^k} - \theta' \frac{\partial n}{\partial \tau^k} > 0$$

Appendix C.2. Proof of Proposition 7

Under the assumption that the tax policy is conceived in such a way to maintain total revenues TR a fixed proportion ζ of GDP, the government budget constraint implies:

$$tr \equiv \frac{TR}{Y} = \tau^k + \frac{1-\gamma}{\gamma} \tau^l = \zeta > 0$$

so that

$$\frac{d\tau^l}{d\tau^k} = -\frac{\gamma}{1-\gamma} < 0
\tag{C.20}$$

We can write the total change of any variable under investigation as follows:

$$\left. \frac{dx}{d\tau^k} \right|_{(\bar{r})} = \frac{\partial x}{\partial \tau^k} + \frac{\partial x}{\partial \tau^l} \left. \frac{d\tau^l}{d\tau^k} \right|_{(\bar{r})},$$

with $x = (l - \theta(n)), n$;

Consequently, from proposition 4 it follows that $\left. \frac{dn}{d\tau^k} \right|_{(\bar{r})} < 0$ and

$$\left. \frac{d(l - \theta(n))}{d\tau^k} \right|_{(\bar{r})} > 0$$

As for the rate of BGP rate of growth, from eqs. (C.13), (C.16) and (C.20) it follows that:

$$\begin{aligned} D \left. \frac{d\bar{r}}{d\tau^k} \right|_{(\bar{r})} &= D \left\{ \left. \frac{\partial \bar{r}}{\partial \tau^k} + \frac{\partial \bar{r}}{\partial \tau^l} \left. \frac{d\tau^l}{d\tau^k} \right|_{(\bar{r})} \right\} =, \\ r \frac{(\Omega + \theta' \omega)}{l - \theta(n)} &\left(\frac{\bar{r}}{r^* - \bar{r} + \bar{z}} \frac{1 - \gamma}{\gamma \sigma} - M \right) + r \Omega \left(-\frac{\Psi''}{\Psi'} + \frac{\Psi'}{\Psi} \right) + \frac{r \omega}{r^* - \bar{r} + \bar{z}} - \frac{\gamma}{1 - \gamma} \frac{1}{1 - \tau^l} \frac{\bar{r}}{l - \theta(n)} (\Omega + \theta' \omega) \\ r \frac{(\Omega + \theta' \omega)}{l - \theta(n)} &\left(\frac{\bar{r}}{r^* - \bar{r} + \bar{z}} \frac{1 - \gamma}{\gamma \sigma} - M - \frac{1 - \tau^k}{1 - \tau^l} \right) + r \Omega \left(-\frac{\Psi''}{\Psi'} + \frac{\Psi'}{\Psi} \right) + \frac{r \omega}{r^* - \bar{r} + \bar{z}} \end{aligned} \quad (C.21)$$

By exploiting (C.17) (C.21) can be written as:

$$D \left. \frac{d\bar{r}}{d\tau^k} \right|_{(\bar{r})} = r \frac{(\Omega + \theta' \omega)}{l - \theta(n)} \left(\frac{1 - \gamma}{\gamma} \frac{\bar{r} - \rho + n}{r^* - \bar{r} + \bar{z}} + 1 - \frac{1 - \tau^k}{1 - \tau^l} \right) + r \Omega \left(-\frac{\Psi''}{\Psi'} + \frac{\Psi'}{\Psi} \right) + \frac{r \omega}{r^* - \bar{r} + \bar{z}}$$

Hence, sufficient for $D \left. \frac{d\bar{r}}{d\tau^k} \right|_{(\bar{r})}$ to be positive is that $\tau^k \geq \tau^l$.